An Egg Today and a Chicken Tomorrow: A Model of Social Security with Quasi-Hyperbolic Discounting*

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Abstract

Strotz (1956) first suggested that people are more impatient when making short-run tradeoffs than long-run ones. Many experimental studies supports this conjecture. Motivated by recent evidence of the British Department of Work and Pension (2006), we apply this framework to retirement decisions. We propose a three-periods OLG model with quasi-hyperbolic consumers who save for post retirement consumption in the first period and choose retirement age in the second. We show that this behavioral assumption explains the observed drop in post retirement consumption due to lack of saving and the high level of voluntary (*i.e.* not due to disability or dismission from the firm) early exit from the labor force. When deciding whether to retire or not, workers weight too much the costs of remaining at work (*i.e.* disutility of working an additional year, implicit tax on continued activity) and too little the benefits of postponed retirement (*i.e.* increase of the Bismarckian component of the pension formula), that are perceived as too far in the future. Moreover, impatient individuals do not save enough for their post-retirement consumption, often regretting for this lack of commitment. We also investigate the implications of time inconsistency for a political economy model in which voters determine simultaneously the size and the degree of redistribution of the pension system. We show that, when voting over the payroll tax, time inconsistent young workers, looking for a commitment device that increases both saving and retirement age, form a coalition with rich in order to *decrease* the size of the system. When voting over the degree of redistribution, they form a coalition with poor individuals to increase the flat part of the pension formula. Our model provides a political justification for the negative relationship between size and redistribution observed in most OECD countries (Disney 2004).

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1 Introduction

The future financial sustainability of social security systems is a central issue in political debates across the world. In most countries the increase of life expectancy has not been followed by a corresponding increase of the minimal retirement age. The problem is strengthen by the fact that most pension systems have implicit or explicit features that allow workers to quit their workplace before this minimal age. The combination of insufficient reforms and generous early retirement provisions[1] has generated severe pension crisis, especially in European countries. It is not surprising, therefore, that governments are trying to deeply reform public pension systems, either by increasing the minimal retirement age or by tightening the link between pension benefits and past earnings. Although necessary, reforming the pension system is a complicated task. For instance, the two reforms proposed above have a very different political appeal: while the former is heavily opposed by workers and unions and thus appear to be politically unfeasible, the latter receives more support, since it introduces "incentives" and "disincentives" that are supposed to induce workers to postpone efficiently and autonomously their retirement age, without any imposition by the government.

Before implementing any reform, it seems appropriate for the policymaker to know exactly the determinants of early retirement. The economic literature (Mulligan and Sala-i-Martin, 1996, and Conde Ruiz and Galasso, 2004) emphasizes the role of firms (the *push* argument): early retirement concerns primary low skilled workers, who have been dismissed by their employer either because they are more likely to receive a negative shock on their productivity level, or because they create a negative externality on young workers. Moreover, as stressed by Gruber an Wise (2000), the pension system itself induces early retirement, through an implicit tax on continued activity[2]. Explanations of early retirement based solely on exogenous factors are not entirely justified from an empirical point of view (DWP, 2006): retiring earlier than planned seems to be an individual choice, and depends on individuals' preferences for leisure (the *pull* argument). Moreover, individuals ex ante overestimate their retirement age, and ex post, after having realized that they had overestimated post retirement savings and pension benefits, they regret about their lack of commitment. Based on these observations, this paper proposes a different view of early retirement, based on the *pull* argument, that complements both the "push" and the "implicit tax" explanations.

More precisely, in order to fill the gap between the perfect rationality assumed so far in Public Economics and Political Economy and the bounded rationality observed in experiments (Kahneman

[1]We define early retirement as the discrepancy between planned and effective age of retirement (Blondal and Scarpetta, 1998).

[2]To better understand the concept of implicit tax, consider, for example, a 59 years-old worker: what is the change in his pension benefits if he retires at age 60 instead of age 59? We call *accrual rate* the difference between the two pensions. If this rate is positive, working an additional year increases the total compensation; if the accrual is negative, working more reduces the pension. Thus a negative accrual rate discourages continuation in the labor force and a positive one encourages continued labor force participation. We have an *implicit tax* on continued activity whenever the ratio of the accrual rate to net wage earnings is negative, otherwise we have an *implicit subsidy*. Gruber and Wise (2000) show that the accrual rate and the associated tax are negative at older ages: continuation in the labor force reduces pension benefits, providing an incentive to leave the labor force earlier.

and Tversky, 1979) and following the recent developments of the Economics and Psychology literature, we introduce hyperbolic discounting[3] in individuals' preferences. Hyperbolic individuals, when facing intertemporal trade-offs, change their preferences over time, such that what is preferred at one point in time far in the future is inconsistent with what is preferred today. An example by Thaler (1981) illustrates this point: if a person has to choose between an apple in 100 days or two apples in 101 days, he will probably prefer the second option. However, proposing the same trade-off between today and tomorrow, if the individual has a high preferences for today's utility, his choice may change and the first alternative becomes the preferred one. This example shows how certain agents are more impatient in the short-term than in the long-run. Laboratory and field studies confirm this intuition and find that discount rates are much greater in the short run than in the long run. It follows that present-biased individuals increase their utility level if a commitment device that force them to stick with the long run plans would have been made available to them (Laibson, 1997).

This paper introduces three sources of heterogeneity among agents: the first two, productivity level and age, are common features of most political economy model of social security (see Galasso and Profeta, 2002, for a review), whereas the third one, the difference in the degree of time inconsistency, is the main peculiarity of this work. In particular, to better match the experimental evidence, we assume that only certain individuals display bounded rationality, with different degrees of awareness.

The reasons for introducing quasi-hyperbolic discounting are three: first, we complement the economic literature on early retirement by showing that the observed early exit from the workforce can be also explained through a model that focus mainly on workers' (bounded) rationality, and not only by exogenous factors. We show that, due to their lack of commitment, hyperbolic individuals are not able to stick with their optimal retirement plans, and when deciding whether to retire or not before the mandatory retirement age (benefiting from the early retirement provision), they weigh too much the costs associated to remain at work (foregone leisure) and less to the benefit implied by a longer working career (increase in pension benefit, if the the formula has a component related to past earnings).

Second, this model provides a behavioral justification for the drop in post retirement consumption due to inadequate saving observed among early retirees (Loewenstein, 1991 and Bernheim, 1995 and Laibson, 1997).

Third, we analyze the implications of hyperbolic discounting for a voting model in which individuals vote over the main characteristics of the pension system (its size and its degree of redistribution). To our knowledge, this is the first paper that studies the implication of this result for a political economy model with endogenous retirement age. By introducing time inconsistency as an additional source of heterogeneity among individuals, our political model is able to provide a more complete explanation for the paradoxical observation that countries with low redistribution in social security systems have also larger public pension expenditures than countries with more redistributive social security systems (Disney,

[3]In the paper, we will refer alternatively to present-biased preferences, time inconsistency and preference reversal. Although in the literature the meanings of these expressions are slightly different, for us all these terms denote a situation where people are not able to commit to future actions, and have a strict preference for the present.

2004, Conde Ruiz and Profeta, 2005). We show that the relevant winning coalition that determines the size of the pension system is not composed simply by all poor and all retirees, as shown in traditional political model of social security (Galasso and Profeta, 2002). More precisely, young hyperbolic workers, also with low productivity levels, prefer to decrease the *decrease* the payroll tax. This counterintuitive result comes from the fact that sophisticated hyperbolic workers (who are aware of their inconsistency issue), looking for a commitment device that increases both their retirement age and saving, form a coalition with rich individuals, who prefer to rely on private savings instead of the public pension. Moreover, when voting over the degree of redistribution, time inconsistent individuals form a coaltion with poor and old, in order to increase the flat part. The intuition goes as follows: besides the traditional intergenerational and intragenerational forms of redistribution, our model introduces a third one, namely from time consistent to hyperbolic individuals. We provide a behavioral justification for the negative relationship between size and redistribution observed in most OECD pension systems: the more time inconsistency represents an issue[4], the smaller will be the pension system, and the higher will be its degree of redistribution. Finally, we use the results of both the theoretical and the political model to evaluate the effectiveness of perspective reforms of the pension system. The policy implications are immediate: since individuals are not able to fully understand intertemporal trade-off, the model suggests that, in spite of the popular view, increasing the minimal retirement age would be a more effective way for the government to overcame the financial crisis of the pension system.

The paper is organized as follows: in section 2 we shortly review of the behavioral literature and its main results; in section 3, we present the behavioral anomalies in retirement decisions (retirement age and saving for retirement) that show the empirical relevance of our hyperbolic model and motivate our research. In section 4, we discuss the relationship between social security and redistribution. In section 5, the basic features of the model are introduced. In section 6 we analyze the optimal choices of a young worker (saving) and of an old one (retirement age). In section 7 we derive the solution of an utilitarian social planner. In section 8, the majority voting equilibrium is determined. Section 9 concludes.

2 Background on Time Inconsistency

The economics and psychology literature (see Laibson, 1997, for a review) has challenged that view that individuals are perfect far-sighted optimizers. Large body of experimental and field evidence has demonstrated that the way individuals take a decision departs substantially form the perfect rationality paradigm. Examples of such departures are given by anxiety, misperceptions, dynamic inconsistency, mistakes, anticipatory feelings, reference points and loss aversion. In particular, starting from Laibson (1997), most of the behavioral literature has focused on a particular form of bounded rationality: time inconsistency in intertemporal decisions. In the hyperbolic model, each individual is represented as a

[4]We are not saying that certain countries are more time inconsistent than others, but only that the existence of provately-provided commitment devices differs across countries: if such instruments are not available, hyperbolic individuals look for other forms of commitment, in the form of a lower payroll tax.

collection of selves: present selves overweight current payoffs compared to future ones, giving rise to a conflict between preferences of different intertemporal selves. More formally, an hyperbolic individuals has the following intergenerational utility function (Strotz 1956, Phelps and Pollacks 1968, Laibson 1997):

$$u_0(.) + \beta \sum_{t=1}^{T} \delta^t u_t(.) \tag{1}$$

where β is the short term psychological discount factor and δ the time-consistent, long term one. The discount discount factor between now and the next period is $\beta\delta$, whereas the discount factor between any two periods in the future is δ or, equivalently, that the discount factor between today and the next period is declining, and constant thereafter.

Following Gruber and Köszegi (2004), we assume that individuals are heterogenous with respect their degree of time inconsistency; in other words, the value of β differs among individuals. We model this heterogeneity as follows: each agent is characterized by a **perceived** psychological discount factor, which may or may not differ from the **true** one. The following definition clarifies this distinction.

Definition 1 *The perceived discount factor is the one individuals have in mind when making decisions. The real psychological discount factor represents their true degree of time inconsistency. The Overconfidence is the difference between the perceived and the true psychological discount factor. More precisely, we have:*

Individual Type	**Perceived DF $\hat{\beta}$**	**True DF**	**Overconfidence**
Exponential	1	1	0
Sophisticated	β (< 1)	β (< 1)	0
Naive	1	β (< 1)	$1 - \beta$

Table II: Perceived and real discount factors.

Exponential, or time consistent, individuals are rational utility maximizers. *Sophisticated* individuals are aware of their time inconsistency (their perceived discount factor coincides with the true one): self t knows that self $t+1$ will want to do something other than what self t would have him to do. Therefore, the best thing self t can do is to find a commitment device that force him to actually follow his (optimal) plan. Finally, *naive* agents are not able to make consistent plans through time (future selves will change systematically those plans) and are not able to actualize predicted or desired future levels of consumption, since their perceive themselves as time consistent, but they are actually hyperbolic.

This paper contributes to the recent literature on applied behavioral economics (Della Vigna and Malmendier, 2004, Della Vigna and Paserman 2005). So far, however, only few papers have considered behavioral and psychological issues into more traditional model of public and political economy. Feldstein (1985), for instance, shows that, in a framework with completely myopic individuals who do not save at all, it may be optimal to have either no social security or one with a very low replacement ratio. Imrohoroglu, Imrohoroglu and Joines (2004) extend Feldstein model, allowing for hyperbolic discounting, and show that social security is a poor substitute for private saving, even for naive consumers with high short term discount parameter. Diamond and Köszegi (2003), introducing endogenous retirement age in Laibson (1997) quasi-hyperbolic consumption model, investigate the effects over saving of a social security system, seen as a commitment device for impatient individuals. Gruber and Köszegi (2004) use the quasi-hyperbolic setting to derive the optimal tax for addictive goods (cigarettes, fatty foods etc.), and find that it includes a "self-control adjustment" component, that provides to sophisticated a commitment device that reduces consumption of such goods. Cremer et al. (2005) study the optimal design of a linear pension scheme in a world of wage heterogeneity and complete naiveté for a group of workers who do not save for retirement. In a companion paper (Cremer et al., 2007), they studies the determination through majority voting of a pension scheme when there are far-sighted and myopic individuals, who tend to look for instant gratification when deciding over saving and labor supply. Their model, however, considers only two groups of consumers, fully myopic who do not save at all and far-sighted, and they do not consider endogenous retirement age, as we do.

Except for our behavioral assumption, our paper contributes also to the literature on the political economy of early retirement (Conde-Ruiz and Galasso 2004, Casamatta et al. 1999); our work is also closely related to Conde-Ruiz and Profeta (2005) and Casamatta et al. (2005)[5].

3 Time Inconsistency in Retirement Decisions

In the two following subsections, we present two stylized facts that confirms the validity of our behavioral assumption. We focus on the discrepancy between planned and real retirement age (subsection 3.1) and the reduction of post-retirement consumption due to a lack of saving (subsection 3.2).

3.1 The Retirement Age

The objective of this subsection is to illustrate the reasons behind the observed high levels of unanticipated retirement.

The literature generally explains the early exit from the workforce with a redundancy/dismission argument (Sala-i-Martin 1996, Ahituv and Zeira 2000, Conde-Ruiz and Galasso 2004): older workers are excluded from the labor force by the firm, either because they have received a negative shocks on their productivity or because they create a negative externality on young's productivity that decrease aggregate output. We refer to this explanation as the *push* argument.

We depart from this view by proposing the *pull* argument: the observed unexpected early exit from labor force is mainly due to voluntary quits and it is a direct consequence of individual, hyperbolic, preferences: when deciding whether retiring or not, present-biased workers do not evaluate correctly the

[5] In this paper, individuals, after having chosen retirement age and post-retirement saving, vote over the payroll tax and the level of the implicit tax on continued activity. They show that a biased pension system may emerge as an equilibrium of the voting game.

costs and the benefits associated to their choice, and often, *ex-ante* overestimate their future retirement age.

To give preliminary evidence of our claims, we present a report by the British Department of Work and Pension[6] (2005), where old workers (starting from the age of 50) are asked to self-report their planned retirement age. We can reasonably consider these reports as the utility-maximizing choice of a rational agent who correctly takes into account the costs and the benefits associated with the retirement age. Afterwards, expectations are compared to the true retirement age. Results are reported in Table III.

Age	**% Planned to retire**	**% Effectively retired**	**% Δ**
<65	25	59	+34
65	45	26	-19
>65	20	9	-11
N/A	10	6	-4

Table III: Planned and effective retirement age[7]. *Source*: DWP (2005)

It is immediate to see that the proportion of workers expecting to retire earlier than the mandatory age is much lower than the proportion of those who actually do. One may argue that the push argument can explain this discrepancy: workers are forced unexpectedly into early retirement, either because their low productivity (this could happen in a more or less explicit way, *i.e.* through bribes, psychological pressure etc.) or illness or disability. The report contradicts this conclusion: only 15% of early retirees reported to be made redundant by the firm (explicitly or implicitly); 35% of the sample reported health as the main reason for early retirement, and more than 50% has retired earlier simply because wanted to do, but they were not able to predict it[8]. Furthermore, the report shows that neither there is a clear pattern in the educational or income level of early retirees, nor financial considerations seem to influence the retirement choice (DWP, page 59), in contrast with the idea that early retirees have low level of human capital and income. According to the DWP, then, early retirement is mainly a voluntary choice, and do not necessarily concern only poor workers[9].

Blondal and Scarpetta (1999) shows that conclusions turn out to be true not only for U.K. but also for the majority of European countries (Figure 1). From the table we see that, except for Finland and

[6]Hereinafter DWP.

[7]The mandatory retirement age is 65 but people can retire (at reduced benefits) starting from 55.

[8]In particular, the main reason for early retirement was, for these workers, the "opportunity of spending more time with a partner or family". (DWP, page 55), thus confirming that leisure in old age receives great weight in retirees' preferences. Another weakness of the push argument is that most countries have implemented legislation aimed to reduce age discrimination on the workplace, making dismissal of old workforce more difficult for the firm. For instance, in 2005, the British Government apporved "The Green Paper", whose objective was to limit age discrimination by employers.

[9]"*The Report shows that, among those who were working, the pull factors were clearly more important than the push factors in determining the decision to retire early"* (DWP, page 56). Another possible explanation is that a change of legislation occurred during the period of the survey has induced workers to unexpected early retirement. Indeed, the "Green Paper", implemented during the Blair administration, has raised the minimum pension for low income pensioners. However, in spite the fact that this reform has make early exit more attractive for this group, the data do not show a clear correlation between income level and voluntary early retirement: the reform had not modify incentives.

	Austria	Belgium	Denmark	Finland	France	Germany	Greece	Ireland	Italy	Luxembourg	Netherlands	Portugal	Spain	Sweden	Switzerland	UK
Dismissed or make reduntant	5,1	3,7	23,4	24,1	10,7	9,5	2,5	8,8	2,0	0,0	7,9	1,0	10,2	30,2	4,9	22,0
A job of limited duration has ended	0,2	0,7	7,0	3,8	1,5	0,7	1,7	3,1	1,7	0,5	0,1	0,4	11,1	8,2	N/A	3,6
Personal of family responsibilities	0,2	1,3	0,2	0,0	0,4	0,6	0,5	1,4	1,2	0,2	1,9	0,0	0,2	2,0	2,2	1,6
Own illness or disability	2,6	7,7	9,5	25,0	7,3	22,9	4,1	15,1	5,2	16,6	15,6	2,1	18,3	7,0	28,9	22,8
Early retirement	49,0	30,6	37,2	0,0	16,9	33,1	5,1	15,9	9,2	29,1	42,9	2,3	13,0	25,9	35,1	14,7
Normal retirement	30,2	19,6	2,3	11,7	38,6	10,9	52,2	12,0	53,4	31,7	0,0	1,2	17,8	12,5	22,8	4,8
Other reasons	12,8	36,4	20,3	35,5	24,5	22,3	33,9	43,7	27,5	22,0	31,6	93,1	29,5	14,2	6,1	30,6

Figure 1: Retired males aged 55-64: Main reasons for leaving last job or business. *Source*: Blondal and Scarpetta, (1998)

Sweden, that experienced big labor market shocks in the 90s, all countries exhibit large share of voluntary early retirees (around 30%).

These stylized facts show that individual are not always farsighted optimizers: even if the have strong preferences for leisure, they should be able to anticipate their retirement age. Therefore, the high level of early retirees can not be explained only by the usual models of disability and/or low human capital.. Our framework tries to justify the discrepancy between planned and effective retirement age not by a redundancy argument but by assuming that individuals, although with different degrees, display of time inconsistency *à la* Laibson: some workers (exponential) planned to retire at a certain age and effectively did it, some others know that their retirement age will be lower than the optimal one (sophisticated) and others simply fool themselves and retire earlier than planned (naive).

3.2 Savings for Retirement

A well known result in intertemporal decision theory is that consumers prefer a smooth or increasing consumption path and save accordingly, even in presence of stochastic income changes (Loewenstein 1991). In reality, however, we observe that individuals consume too much and save too little for their post retirement consumption[10]. Loewenstein (1991) shows that individual consumption paths are actually declining, even in absence of liquidity constraints: retirees therefore experience a substantial reduction of their consumption levels. The sharp drop in the average consumption around the typical retirement age is due to negative innovations to the income process because workers have overestimated their retirement income and saving (Bernheim et al., 1988).

Haussman and Paquette (1987) show that the drop in post retirement consumption is more severe for early retirees. According to Bernheim (1995) these behaviors can not be explained through the standard life cycle framework but are more likely justified by psychological issues, and in particular by psychological impediments to adequate planning for post retirement consumption. This intuition is confirmed by

[10] The problem is particularly severe in U.K., as stressed by British newspapers: "Not enough of British are saving for retirement and many of those woh are investing for the future are underestimating the cost of retirement ". (*The Guardian,* November 17, 2005). "The gap between what the nation should be saving each year for our retirement and what we are actually putting away is £27bn" (*Association of British Insurers*, 2005). "44% of the workforce (12 million workers) did not have pensions provision beyond those on offer form the state" (*Adair Turner, president of Britsh Pensions Commission,* 2005).

Laibson and al. (1998): not only individuals save too little, but also regret for this choice: 76% of U.S. workers near the retirement age believe that they should have saved more. The quasi-hyperbolic model (Laibson et al. 1998) provides a theoretical justification for these behaviors: simulations in Laibson and Harris (2001) show that hyperbolic calibrated simulations are able to reproduce the observed high comovement between consumption and income and the drop in post retirement consumption better than exponential calibrated simulations.

A remark is necessary: recently, in order to boost individual savings and to provide to time inconsistent individuals commitment devices, new financial instruments were created (Benjamin, 2003). In the U.S., for instance, these instruments took the form of tax-deferred savings accounts, such as IRA, 401*(k)*,403*(b)*, 457*(b)* and 457*(f)* etc. In our model, however, we assume that markets are incomplete, in the sense that instruments helping hyperbolic individuals to both save more and retire later are not available. This assumption could appear unrealistic but can be easily justified: first, there is no sure evidence that the introduction such plans has effectively boosted individual savings (Bernheim, 1999). Second, assuming the completeness of financial markets requires also that we should take into account "counter-commitment devices" that exploit consumers' bias towards the present by reducing the commitment power of deferred saving accounts. Indeed, in the U.S., the introduction of tax deferred saving account was followed by the boom of revolving credit cards.

4 Social Security and Redistribution

One of the main aims of a PAYG system is the ability to redistribute income both within and between generations[11]. Depending on link between benefits and individual contributions, pension systems are classified as *Bismarckian, where benefits are proportional to contributions* or *Beveridgean, where benefits are flat* Since contributions paid are proportional to earnings, the former implies, *ex-ante*, less intergenerational redistribution than the latter. Counterintuitively, more redistribution does not necessarily imply that the share of GDP devoted to pension is higher; as shown by Conde-Ruiz and Profeta (2005), there exists a negative correlation between the degree of intragenerational redistribution and the size of the PAYG system: Bismarckian systems tend to be bigger than Beveridgean pension schemes, in spite of the fact the latter is explicitly designed to redistribute income and thus it should receive more support from low income individuals.

A natural question arises: are Beveridgean systems effectively able to redistribute income from who earned more in their working years to those who earned less? Some papers have recently tried to answer this question, with particular emphasis on the (Beveridgean) U.S. Social Security.

Liberman (2001) shows that, at the within-cohort level, redistribution is not related to lifetime income but to other factors: it redistributes not from rich to the poor, but from people with low life expectancies

[11] A pensions system serves two other fundamental functions: it forces individuals to save for post retirement consumption and insures workers against disability risks.

to people with low life expectancies, from single workers and from couples with high earnings by the secondary earner to one-earner couples and, finally, from workers with an earning history longer than 35 years to workers who concentrate their earnings in 35 or fewer years. Moreover, not only redistribution seems not to be related to income factors, but also, in some cases, the system appears to be regressive: for example, 19% of workers in the top lifetime *income* quantile receive net transfers that are greater than the average transfer for people in the lowest lifetime *income* quantile.

Coronado, Fullerton and Glass (2000) show that the claimed progressiveness of the pension system depends on a series of simplifying assumptions: looking only at redistribution on an annual basis (from workers to retirees), the system appears to be highly progressive. But, when more realistic assumptions are taken into account (different mortality probabilities etc.), the progressiveness of the system decreases and it may even become negative. This conclusion seems to be valid not only for the U.S. but also for other social security systems across the world: some Beveridgean systems have features that reduce the explicit level of redistribution. The reverse appears also to be true: Disney (2001) show that also some Bismarckian systems redistribute income from rich to poor, in spite of the fact that they are modeled to guarantee high replacement rates.

These observations allow us to classify OECD pension systems into four groups, on the basis of type (Bismarckian and Beveridgean), the level of redistribution (high or low), and the generosity, expressed as a fraction of GDP devoted to pensions. The first classification(Disney 2001) is made on the basis of the replacement rates guaranteed by the pension system (higher replacement rates make the system more Bismarckian), whereas the second is calculated taking into account the progressivity index[12] calculated by OECD (2005).

I - Bismarckian **High Redistribution** **High Expenditure**	**II - Bismarckian** **Low Redistribution** **High Expenditure**
Belgium, Austria Germany	Greece Italy
III - Beveridgean **High Redistribution** **Low Expenditure**	**IV - Beveridgean** **Low Redistribution** **Low Expenditure**
Canada, UK Denmark, New Zealand	US, The Netherlands Japan, Switzerland

Table IV: Classification of pension systems of OECD countries.

Political economy models of social security have tried to justify these cross-countries differences in terms of generosity, degree of redistribution, choice of the type of the system, either from an economic

[12]The index is based on the Gini coefficient and it considers only mandatory parts of public pension programs. We consider Beveridgean and Bismarkian pension system with progressivity index above, respectively, 60% and 20%, as "highly redistributive". See Table V in the Appendix for details.

or a behavioral standpoint. The first group of paper appeals to the political power of the lower class, that is decisive in the political process and is able to determine the main characteristics of the system (generosity, early retirement provision etc.). Meltzer and Richards (1982) and Tabellini (1992) show that the equilibrium size of the pension system is determined by a coalition of old people and young poor workers.

Conde-Ruiz and Galasso (2003, 2004) and Sala-i-Martin (1996) show that an early retirement provision is introduced in the system by a coalition of old workers with incomplete working history and low-income young. The first group, dismissed by their firm because of a negative productivity shock has came, favors early retirement. The second group agrees to introduce an early retirement clause since they are likely to become, in the future, early retirees. Conde-Ruiz and Profeta (2005) propose a political economy model in which individuals vote over the size and degree of redistribution of the pension formula. It is shown that a Beveridgean system is supported by low-income agents, who gain from its redistributive feature, and rich individuals, who seek to minimize their tax contribution and to invest their resources in a private pension scheme.

A second group of papers considers behavioral and psychological factors to explains such differences. Benabou and Ok (2001) show that poor may not support high levels of redistribution today because they hope to be rich in the future. Alesina and Angeletos (2005) and Benabou and Tirole (2005) develop models that consider explicitly fairness and equality of opportunities in individuals' preferences: redistribution is low when economic success is perceived as driven by effort and not by luck. Finally, other papers (see Alesina and La Ferrara 2005 for a review), consider identity: individuals care about who benefit from redistribution (for instance, they do not want to subsidize people from different ethnic groups). Thus, very heterogeneous societies may have lower preferences for redistribution.

This paper merges behavioral and economic explanations by assuming, at the same time, heterogeneity in productivity levels, age and degree of time inconsistency. Our economic model show that some agents, independently of their income, prefers to anticipate the age of retirement and prefer to consume more when young instead of saving for post-retirement consumption. Furthermore, our political model, in which the size of the social security system and the degree of redistribution are chose by direct majority voting, shows that a winning coalition of hyperbolic individuals is able to determine both the generosity and the degree of redistribution of the PAYG system. In particular, our model explains why low level of redistribution are often associated with big pension programs: hyperbolic young workers, looking for privately-provided commitment devices that increase both saving and retirement age, form a coalition with the rich in order to *decrease* the payroll tax. On the other hand, when voting over the degree of the redistribution of the PAYG system, the winning coalition includes poor and hyperbolic individuals both in favor of a more redistributive system.

5 Social Security, Retirement and Time Inconsistency: A Model

In this section we present a model of intertemporal consumption with endogenous retirement age (as in Casamatta et al. 2005 and Conde-Ruiz and Galasso 2004), with the assumption of quasi-hyperbolic preferences.

5.1 Basic Setup

Timing The model is set in discrete time There are three periods and two generations, *young* and *old*. At period 1, an individual is young: he supplies inelastically labor and saves for post-retirement consumption. At period 2 (*pre-retirement*), an individual is old: he supplies labor (but with an effort cost); we interpret labor supply as the choice of the age of retirement, as in Casamatta et al. (2005). In period 3 (*post-retirement*), the individual is retired and consumes accumulated savings and a pension transfer P[13]. Moreover, he enjoys some additional leisure, which is inversely related to the retirement age. The length of period 1 is normalized to 1, whereas the length of periods 2 and 3 are endogenous, since they both depend on the retirement choice of the agent. The total length of period 2 and 3 is normalized to 2.

Heterogeneity Besides age, individuals differ for their degree of (perceived) time inconsistency and their productivity level. For the former, we refer to Table III and we consider three type of agents: exponential, sophisticated and naive, who differ on the basis of their short term discount factor, $\hat{\beta}_j$. The the long run discount factor, δ, is the same for all individuals. For the latter, we assume that:

- **A1**: Each worker is assigned with a random productivity level ω, distributed on the support $[\omega_-, \omega_+]$ according to a density function $f(.)$ and a c.d.f. $F(.)$. The median productivity, ω^M, is lower than the mean, $\bar{\omega}$.

For simplicity, we assume that both the productivity level and the behavioral type remain unchanged across periods. We denote with n the exogenous rate of population growth; N^r is the number of old retirees in the economy, N^o the number of old workers and $N^y = (1+n)(N^o + N^r)$ the number of young. The total population is therefore $N = N^y + N^o + N^r$ or, equivalently, $(N^o + N^r)(2+n)$. Each behavioral type represent a fraction of the whole population, such that $N^i = N^{i,e} + N^{i,n} + N^{i,s}$, with $i = y, o$, where the subscripts e, n and s stand for exponential, naive and sophisticated individuals[14]. None of them represent 1/2 of the population.

[13]Savings can be interpreted as a voluntary payment to a integrative pension plan, whose benefits are paid only when the beneficiary is retired. The fact that $P(\widehat{z_j})$ is paid only after retirement, is indeed a realistic assumption; Sala-i-Martin (1996), shows that, for 70 out of 108 countries where this information is available, the elderly must show that they do not get labor income from any other source in order to collect old age pensions.

[14]Notice that we the distribution of behavioral types among old retirees does not matter for our purposes, since all economic decisions are already taken.

Utility The multi-selves formulation of expression (1) implies that each individual of type $j = e, n, s$ maximizes the following intertemporal utility function:

$$U\left(c_y^j, c_o^j\right) = u(c_y^j) + \hat{\beta}_j \delta u(c_o^j) \tag{2}$$

where $u(.)$ is the instantaneous utility function, which is assumed to be the same for all individuals, increasing and concave, and satisfying Inada conditions. We denote c_y consumption when young and c_o the consumption when old (second and third periods). Each worker (young and old) pays a proportional payroll tax $\tau \in [0, 1]$ on his wage. Consumption when old is net of the disutility of working when old, $m(z)$ and the utility from leisure when retired, $l(z)$, both expressed in unit of consumption. $z \in [0, 1]$, as in Casamatta et al. (2005), denotes the fraction of the second period an old agent spent working. It follows that the lengths of periods 2 and 3 are, respectively, z and $(2 - z)$. We assume specific form for functions $m(z)$ and $l(z)$:

- **A2**: $m(z) = \frac{\gamma z^2}{2}$, where γ measures the intensity of the disutility of effort.
- **A3**: $l(z) = \frac{\psi(1-z)^2}{2}$, where ψ represents the intensity of the utility from leisure. We assume that $\gamma > \omega_+ > \omega_- > \delta\psi$.

Consumption levels for young and old of type $(\omega,\, j)$ are then given by:

$$\begin{aligned} c_y^j &= (\omega(1-\tau) - \widehat{s_j}) + \hat{\beta}_j \delta \left(c_o^j\right) \\ c_o^j &= \underbrace{\omega \widehat{z_j}(1-\tau\theta) - \frac{\gamma\left(\widehat{z_j}\right)^2}{2}}_{\text{Pre-Retirement Consumption}} + \hat{\beta}_j \delta \underbrace{\left(P(\widehat{z_j}) + \frac{\psi(1-\widehat{z_j})^2}{2} + (1+r)\widehat{s_j}\right)}_{\text{Post Retirement Consumption}} \end{aligned} \tag{3}$$

where $\widehat{s_j} \geq 0$ and $P(\widehat{z_j})$ represents, respectively, saving and pension benefits, that depends on the retirement age, $\widehat{z_j}$. The "hat"stresses the fact that , due to our behavioral assumption, consumption levels, retirement age and saving depend all on the perceive discount factor $\hat{\beta}_j$. Finally, let r the exogenous interest rate paid on accumulated saving; to simplify notation, we impose $1/(1 + r) = \delta$.

From the intertemporal utility function and the old budget constraint, it is easy to see that individuals are not only time inconsistent *between* generations (equation 2) but also *within* generations (equation 3). It follows that the implied discount factor from the point of view of a young individual who is evaluating the trade-off between working and retiring in the old age is δ, whereas for an old individual is $\hat{\beta}_j\delta$. This assumption captures the essence of time inconsistency: when deciding over the retirement age, old workers may change their preferences, giving too much weight to the present costs of staying at work (the disutility of effort), and less to the future benefits (increase of pension benefits). Moreover, hyperbolic young, when choosing saving, discount the post-retirement period by the factor $\hat{\beta}_j\delta^2$.

The Pension System The PAYG system is assumed to be balanced every period, so the sum of all awarded pensions is equal to the sum of all contributions paid. The pension received by an individual of type (ω, j) is given by:

$$P(z) = \underbrace{\alpha\tau\omega(1+n+\theta z)}_{I} + \underbrace{(1-\alpha)\tau(1+n)\bar{\omega}}_{II} \tag{4}$$

To introduce distributional concerns in our model, we assume that the formula includes two components[15]: a *Bismarckian* (*I*, related to contributions paid by the worker and its retirement age) and a *Beveridgean* one (*II*, flat, related to the mean wage of the economy, $\bar{\omega}$); both components have a rate of return equal to rate of population growth, n. The parameter $\theta \in [0,1]$ represents the implicit tax on continued activity (Gruber an Wise, 2000), and takes into account that the pension system itself induces workers to leave earlier their job: for $\theta = 0$, the pension system is neutral, *i.e.* the marginal benefit of working one more year is exactly ω. For $\theta > 0$, individuals have incentive to retire earlier[16], since the marginal benefit of working more is only $\omega(1-\theta\tau)$.

The weight $\alpha \in [0,1]$ in (4) is the Bismarckian factor (Conde Ruiz and Profeta, 2005). If $\alpha = 1$, the system is purely Bismarckian, and benefits are proportional to the contributions paid. For $\alpha = 0$, the system is purely Beveridgean: benefits are flat and not related to the worker's wage history. If $\alpha < 1$, the pension has two tiers: a flat one, which provides a minimum amount of income, and a second one that relates pension benefits to the history of previous wage earnings.
The implicit tax on continued activity, θ, For the rest of the paper, we make two technical assumptions about α; these assumptions will guarantee the existence of our political equilibrium.

- **A4**:

$$\alpha \leq \frac{1}{(1+\theta)\delta} \quad \text{and} \quad \alpha \leq \frac{1}{1+n}$$

The first inequality implies that θ is not too high[17]. The second inequality, *i.e.* $\alpha \leq \frac{1}{1+n}$ is reasonable, given the low rate of population growth ($n \approx 0$) observed in most industrialized countries.

The Political Process Our voting model works as follows: elections take place every period. Young and old (at the beginning of period 2) vote simultaneously over the payroll tax τ and the Bismarckian factor, α[18]. Time inconsistency affects voting behavior: from one hand, we assume that young do not fully internalize how present vote will affect their future utility, and thus vote having in mind their perceived discount factor, $\hat{\beta}_j$. This is intuitive also for a naive young, who believes that his choices are

[15] The dependence of $P(z)$ on ω is justified by Sala-i-Martin (1999): in most countries, benefits are typically an increasing function of the workers' previous wage history

[16] Cremer, Lozachmeur and Pestieau (2004) show that such distortions are optimal in a second-best setting with a government concerned by redistributive issues and asymmetric information on individuals' productivity and health status.

[17] According to Gruber and Wise (1997), in a sample of 11 developed countries, the value of θ is approximately around 50%, making $\theta \in [1/2, 1]$.

[18] We assume, without loss of generality, that old retirees do not vote. The reason for our assumption is the following: retirees' objective is to increase their consumption as much as possible: thus, *all* retirees will vote for the maximum payroll tax, $\tau = 1$. Moreover, when voting over α, all agents with $\omega < \bar{\omega}$ prefer a completely flat pension, $\alpha = 0$, whereas all individuals with $\omega > \bar{\omega}$ vote for $\alpha = 1$. Since we assume that the income distribution is skewed to the left, the second group will always represent a minority. Therefore, considering that also retirees vote would change only the level of equilibrium functions τ^{mv} and α^{mv}, but not their shape.

optimal, given the incentives he faces. On the other hand, old (naive) realize *ex-post* their true β and their oveconfidence, which has lead to save and retire suboptimally. Therefore, they vote according to their true β, after having made their choice on the basis of the perceived discount factor. Formally, this assumption implies that:

- **A5:** Old's indirect utility function $V^o(\tau_j^o, \alpha; \beta_j, \omega)$ and old's voting behavior depend on on true discount factor β_j. Young's indirect utility function $V^y(\tau_j^y, \alpha; \hat{\beta}_j, \omega)$ and voting behavior depend on the perceived discount factor, $\hat{\beta}_j$.

Because of A5, we have that the total number of time consistent young is equal to the number of naive and exponential: $N^{y,TC} = N^{y,e} + N^{y,n}$; for old, we have: $N^{o,TC} = N^{o,e}$.

Figure 2 summarizes the basic features of our model.

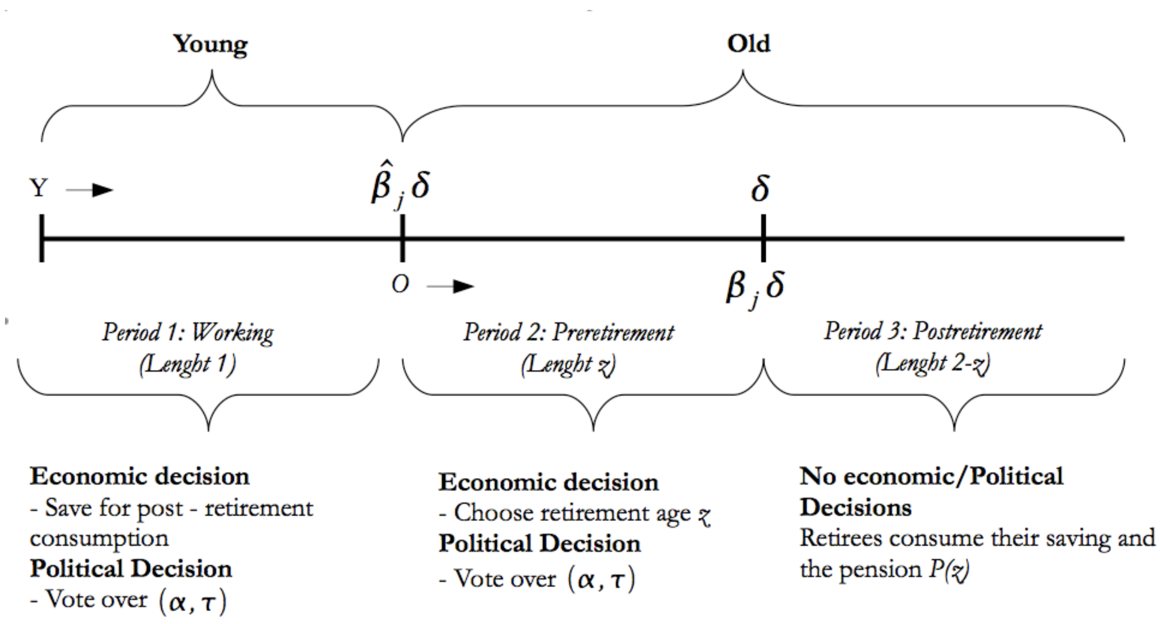

Figure 2: Timing of the Model

6 The Optimization Problems

In this section we characterize optimal choices for young (saving) and old (retirement age).

6.1 Choosing Retirement Age

A type (ω, j) old solves the following program:

$$\max_{\widehat{z}_j} \left[\omega \widehat{z}_j(1-\tau\theta) - \frac{\gamma(\widehat{z}_j)^2}{2}\right] + \hat{\beta}_j \delta \left[P(\widehat{z}_j) + \frac{\psi(1-\widehat{z}_j)^2}{2} + (1+r)s^j\right] \quad (5)$$

subject to:

$$0 \leq \widehat{z_j} \leq 1$$

Notice that an old worker discounts post retirement consumption by $\hat{\beta}_j\delta$. The first order condition associated with problem (5) is:

$$\omega(1-\tau\theta) - \gamma\widehat{z_j} + \hat{\beta}_j\delta\left[\alpha\tau\omega\theta - (1-\widehat{z_j})\psi\right] = 0$$

After some rearrangements, optimal retirement age is given by:

$$\widehat{z_j} = \frac{\omega(1-\tau\theta) + \hat{\beta}_j\delta(\alpha\tau\omega\theta - \psi)}{\gamma - \hat{\beta}_j\delta\psi} \quad \forall j = e, s, n \tag{6}$$

The solution is interior $\leftrightarrow \omega^+\left[1-\tau\theta(1-\hat{\beta}_j\delta\alpha)\right] \leq \gamma$, as implied by A3. All individuals choose to work in the second period, except if $\tau\theta = 1$ *and* $\alpha\omega = \psi$. The intuition is the following: nobody work when the system is completely non neutral and when the increase in the Bismarckian part of the pension due to an additional year spent working ($\alpha\omega$) is exactly equal to the marginal benefit of quitting the job at the beginning of the old age (ψ).

Comparative statics on (6) leads to the following lemma.

Lemma 1 *Optimal perceived retirement age, $\widehat{z_j}$ is:*
(i) increasing in ω, θ and ψ;
(ii) decreasing in payroll tax, τ, and increasing in the Bismarckian factor, α;
(iii) increasing in $\hat{\beta}_j$.

Result *(i)* is in line with the literature (Casamatta et al. 2005 and Conde-Ruiz and Galasso 2004): rich workers, having a lower price of consumption/leisure in period 2 than poor, retire later. In part *(ii)* we show, first, higher payroll taxes decrease perceived retirement age:

$$\frac{\partial\widehat{z_j}}{\partial\tau} = -\frac{\omega\theta(1-\hat{\beta}_j\delta\alpha)}{\gamma - \hat{\beta}_j\delta\psi} < 0 \tag{7}$$

and then that increasing the Bismarckian factor increase the retirement age:

$$\frac{\partial\widehat{z_j}}{\partial\alpha} = \frac{\omega\theta\hat{\beta}_j\delta\tau}{\gamma - \hat{\beta}_j\delta\psi} > 0$$

Notice that this effect is more marked for individuals with high $\hat{\beta}_j$: intuitively, individuals who perceive themselves as time consistent think they can work more if the pension is more earning-related.

More interesting is result *(iii)*: $\partial\widehat{z_j}/\partial\hat{\beta}_j > 0$ implies that the higher is the worker's perceived degree of time inconsistency, the later he *believes* he will retire. This conclusion is in line with the stylized facts presented before: some workers anticipates correctly their retirement age, whereas others overestimate it. Depending on the behavioral type, three cases are possible.

Exponential These individuals have $\hat{\beta}_e = \beta = 1$. Replacing the true discount factor in (6), the retirement age for an exponential worker ($\widehat{z_e}$) is:

$$\widehat{z_e} = \frac{\omega(1-\tau\theta) + \delta(\alpha\tau\omega\theta - \psi)}{\gamma - \delta\psi} \tag{8}$$

which also coincides with the actual one, $\widehat{z_e} = z_e$. In the following, we refer to z_e as the *normative retirement age*, defined as the retirement age to which a young individual would commit himself.[19].

Sophisticated They are aware of their time inconsistency: their perceived discount factor is equal to the real one: $\hat{\beta}_s = \beta < 1$; planned $\widehat{(z_s)}$ and actual retirement age, (z_s) coincide:

$$z_s \equiv \widehat{z_s} = \frac{\omega(1-\tau\theta) + \beta\delta(\alpha\tau\omega\theta - \psi)}{\gamma - \beta\delta\psi} \tag{9}$$

For these agents, however, the lack of commitment leads to a welfare loss; they would be better, in a Pareto sense, if their retirement age would be the one preferred by exponential, as shown in the following proposition.

Proposition 1 *For sophisticated workers, the normative retirement age, z_e, is greater than the equilibrium retirement age z_s. If β is not too small, switching from z_s to z_e will create a Pareto-improvement.*

Proof. All Proofs are in Appendix B. ■

Naive Their true discount factor, $\beta(< 1)$, is lower than the perceived one, $\hat{\beta}_n(= 1)$. Their planned retirement age ($\widehat{z_n}$) is:

$$\widehat{z_n} = \frac{\omega(1-\tau\theta) + \delta(\alpha\tau\omega\theta - \psi)}{\gamma - \delta\psi} \tag{10}$$

but the true one is:

$$z_n = \frac{\omega(1-\tau\theta) + \beta\delta(\alpha\tau\omega\theta - \psi)}{\gamma - \beta\delta\psi} \tag{11}$$

Thus, naive agents exhibit *overconfidence*, which is given by:

$$\Delta z \equiv \widehat{z_n} - z_n = \frac{(1-\beta)\delta[\psi\omega(1-\tau\theta) + \gamma(\alpha\tau\omega\theta - \psi)]}{(\gamma - \delta\psi)(\gamma - \beta\delta\psi)} > 0 \tag{12}$$

Notice that the overconfidence effect justify why we some individuals systematically overestimate their retirement age: they believe will retire at $\widehat{z_n}$, but their bias will lead them to retire at z_n. Moreover, it is easy to see that $\partial\Delta z/\partial\beta < 0$: more time consistent individuals display less overconfidence; in the limiting case, $\Delta z = 0$ if $\beta = 1$.

[19]Rigorously, we should define normative retirement age as the z_j to which self 0 *and* all future selves would commit. However, there is no loss in generality to use this definition of normative retirement age (see Laibson 1998).

6.2 Choosing Savings

Young of type (ω, j) decide how much to save for post retirement consumption anticipating that retirement age will be (6). The maximization program is the following:

$$\max_{\widehat{s_j}} u(\omega(1-\tau)-\widehat{s_j})+\hat{\beta}_j\delta u\left[\omega\widehat{z_j}(1-\tau\theta)-\frac{\gamma(\widehat{z_j})^2}{2}+\delta\left(P(\widehat{z_j})+\frac{\psi(1-\widehat{z_j})^2}{2}+(1+r)\widehat{s_j}\right)\right] \tag{13}$$

subject to:

$$0 \leq \widehat{s_j} \leq \omega(1-\tau)$$

The first order condition associated with an interior solution of $\widehat{s_j}$ is:

$$-u'\left(c_y^j\right)+\hat{\beta}_j\delta^2 u'\left(c_o^j\right)(1+r)=0 \tag{14}$$

Since we assume $(1+r)\delta = 1$, saving are implicitly defined by:

$$u'\left(c_y^j\right)=\hat{\beta}_j\delta u'\left(c_o^j\right) \tag{15}$$

It follows that, $\forall j$, $c_y^j > c_o^j$, since $\hat{\beta}_j\delta < 1$ and the concavity of the utility function: hyperbolic discounting leads to overconsumption in the young age. Individuals are not able to smooth consumption over time and the lower is the true β, the more relevant is this effect[20]. In the following, we will refer to s_e as *normative saving*, *i.e.* saving chosen by an exponential individual. The following proposition shows that sophisticated would increase their welfare if a commitment device that helps them to save s_e is made available.

Proposition 2 *For sophisticated, the normative saving s_e are greater than equilibrium saving s_s. If β^s is not too small, switching from s_s to s_e will induce a Pareto-improvement.*

Proof. See Appendix B. ■

7 The Planner's Problem

Although the aim of this work is mainly positive, it is worth to compare the equilibrium that emerges in the political process with the solution a utilitarian social planner would choose. We restrict policies in the same way as in the voting process considered in the next section: instruments are limited to α and τ. In our framework the planner's problem is not standard: in general, according to Kahneman (1994), each individual maximizes his "decision utility", *i.e.* the utility function that reflects his choices, whereas the government maximizes the agent's "experience utility", the utility function that reflects his welfare; usually, the two concepts naturally coincides, but here they differ, since the planner and the individual do not agree on the weight attributed to future utilities: we have seen that naive individuals are biased toward the present when deciding their saving and their retirement age.

[20]This Euler equation reduces to the one in Casamatta et al. (2005) for exponential and naive ($\hat{\beta}_e = \hat{\beta}_n = 1$), and to the Hyperbolic Euler Equation of Laibson and Harris (2001) for sophisticated ($\hat{\beta}_s = \beta$).

7.1 First Best

In a first best setting, time inconsistency does not represent an issue. The planner solves:

$$\max \int_{\omega_-}^{\omega_+} u(c_y(\omega)) + \delta u \left[x(\omega) - \tfrac{\gamma(z(\omega))^2}{2} + \delta\left(\tfrac{\psi(1-z(\omega))^2}{2} + (1+r)s^y\right)\right] f(\omega)d\omega$$

subject to:

$$\int_{\omega_-}^{\omega_+} \left[c_y(\omega) + \frac{x(\omega)}{1+n} - \frac{\omega}{1+n}\left(1+n+z(\omega)\right)\right] f(\omega)d\omega = 0 \tag{16}$$

where $x(\omega) = \omega z(\omega)(1-\tau\theta) + \hat{\beta}_j \delta \left(P(z(\omega)) + (1+r)\widehat{s_j}\right)$ is consumption when old. Notice that we are considering the true retirement age, $z(\omega)$ since, in a first best setting, we suppose that the government is able to force individuals to retire at the optimal age. From the first order conditions, we have:

$$\begin{aligned} u'(c_y(\omega)) &= \delta(1+n)u'(c_o(\omega)) = \lambda \\ z(w) &= \frac{\omega - \delta\psi}{\gamma - \delta\psi} \end{aligned}$$

where λ is the Lagrange multiplier associated to the budget constraint. The first FOC implies that consumption is equated across individuals and across time, if $\delta(1+n) = 1$. The second condition tells us that optimal labor supply in second period is such that the net marginal disutility of working an additional year, $(\gamma - \delta\psi)z(w)$, equates the marginal benefit of continued activity, ω, net of the forgone leisure $\delta\psi$ enjoyed if the worker would have retired. This first best can be decentralized through a system of lump sum taxes and transfers: however, given the fiscal instruments we are restricting to, this solution can not be achieved.

7.2 Second Best

In the second best setting we assume that time inconsistency is not observable, in the sense that the planner knows only the distribution of naive and sophisticated in the economy[21]. The planner's objective is to set τ and α as to minimize the welfare loss resulting from overconfidence. More precisely, the planner is paternalistic, in the sense of Rabin and O'Donoughe (2001, 2007) and Gruber and Köszegi (2004): the social welfare function is time consistent ($\beta^j = 1$), but the budget constraint take into consideration that hyperbolic individuals retire according to their true discount factor.

7.2.1 The Government's Budget Constraint

Let us define the *perceived* pension, $P(\widehat{z_j})$, as the benefits each type think to receive after retirement, which differs from the *true* one, $P(z_j)$. The perceived function is also the pension young individuals of

[21] Notice that we are implicitely assuming that the planner can not, in the second best, force individuals to retire at the optimum retirement age.

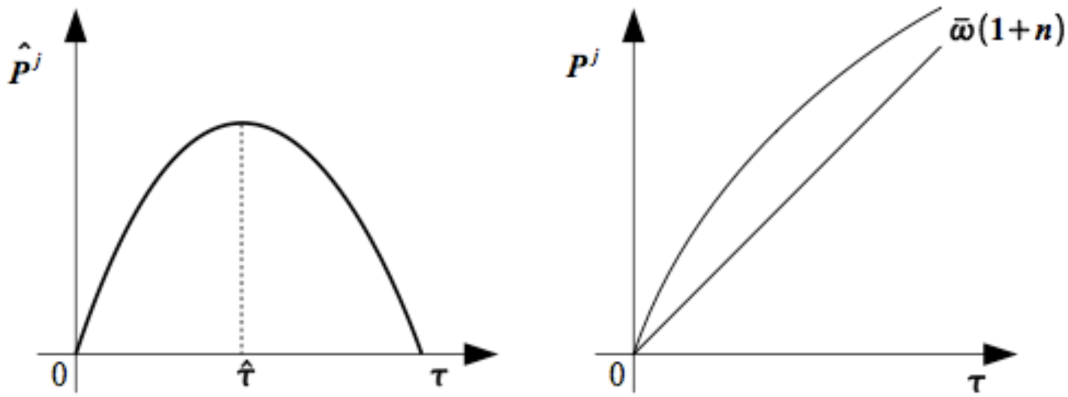

Figure 3: The perceived (left) and the true (right) pension function

type $j = e, s, n$ have in mind when they cast their vote over τ and α. Replacing (6) into (4), we get an expression for $P(\widehat{z_j})$:

$$P(\widehat{z_j}) = \left(1 + n + \frac{\theta\omega(1-\theta\tau) + \hat{\beta}_j\theta\delta(\alpha\tau\omega\theta - \psi)}{\gamma - \hat{\beta}_j\psi\delta}\right)\alpha\tau\omega + (1+n)\tau(1-\alpha)\bar{\omega} \tag{17}$$

Whereas for sophisticated and naive we have that $P(\widehat{z_s}) = P(z_s)$ and $P(\widehat{z_e}) = P(z_e)$, for naive $P(\widehat{z_n}) > P(z_n)$. More precisely, the difference between the two pensions is:

$$\Delta P \equiv P(\widehat{z_n}) - P(z_n) = \frac{\theta\delta(1-\beta)(\omega(1-\theta\tau)\psi + \gamma(\alpha\tau\omega\theta - \psi)}{(\gamma - \beta\psi\delta)(\gamma - \psi\delta)} > 0$$

The next Lemma establishes an important property of the perceived pension function.

Lemma 2 *The pension function is concave in τ, with a maximum in $\hat{\tau} = \frac{\theta\alpha\omega^2 + (1+n)(\gamma - \hat{\beta}_j\psi\delta)(\alpha\omega + (1-\alpha)\bar{\omega})}{2\theta^2\alpha\omega^2(1-\hat{\beta}_j\delta\alpha)}$.*

Proof. See Appendix B. ■

Increasing τ has two effects on the pension function: *(i)* for given $z_j(\hat{\beta}_j)$, it increases the future pension; *(ii)* it reduces the worker's retirement age. The lemma shows that, for $\tau \leq \hat{\tau}$, the first effect prevails. Because of the overconfidence effect, $P(\widehat{z_j})$ differs from the pension scheme that satisfies the government's budget constraint: the discrepancy between the two is due to the negative externality exerted by naive individuals on exponential and sophisticated. The former do not internalize that retiring earlier than planned leads to a drop in total tax proceeds that, consequently, reduce also total benefits.

$$\sum_j N^{o,j} \int_{\omega_-}^{\omega_+} P(z_j)dF(\omega) = N^y\tau \int_{\omega_-}^{\omega_+} \omega dF(\omega) + N^{o,e}\tau\theta \int_{\omega_-}^{\omega_+} \omega z_e dF(\omega) + N^{o,s}\tau\theta \int_{\omega_-}^{\omega_+} \omega z_s dF(\omega) + \tag{18}$$
$$N^{o,n}\tau\theta \int_{\omega_-}^{\omega_+} \omega z_n dF(\omega)$$

Taking into account that naive and sophisticated retire according to (9) and (11)), the average pension for each group of retirees is:

$$\bar{P}_j(\tau,\alpha) = (1+n)\bar{\omega}\tau + \frac{\tau\theta}{\gamma-\beta_j\delta\psi}\left[(1-\tau\theta)E(\omega^2)+\beta_j\delta\alpha\tau\theta(E(\omega^2)-\psi)\right] \text{ for } j=n,s$$
$$\bar{P}_e(\tau,\alpha) = (1+n)\bar{\omega}\tau + \frac{\tau\theta}{\gamma-\delta\psi}\left[(1-\tau\theta)E(\omega^2)+\delta\alpha\tau\theta(E(\omega^2)-\psi)\right] \tag{19}$$

where $E(\omega^2) = \int\limits_{\omega_-}^{\omega_+}\omega^2 dF(\omega)$ and $\beta_j = \{\beta, 1\}$. In the first period, the tax base is fixed, $(1+n)\bar{\omega}$, and depends on the true retirement age in the second period. Differentiating twice (19), we get:

$$\bar{P}'(\tau) = (1+n)\bar{\omega} + \frac{(\theta-2\theta^2\tau)E(\omega^2)+2\beta_j\delta\alpha\tau\theta^2(E(\omega^2)-\psi)}{\gamma-\beta_j\delta\psi} > 0$$
$$\bar{P}''(\tau) = -\frac{2\theta^2E(\omega^2)(1-\beta_j\alpha\delta)-\beta_j\delta\alpha\tau\theta^2\psi}{\gamma-\beta_j\delta\psi} < 0$$

The budget curve, represented in the Figure 3, is concave, always above the line $\tau(1+n)\bar{\omega}$ and equal to it when $\tau\theta = 0$ or $\tau\theta = \frac{E(\omega^2)-\beta_j\delta\alpha\psi}{E(\omega^2)(1-\beta_j\alpha\delta)}$.

7.2.2 Second Best Levels of α and τ

Second best value for α and τ are the solution to the following maximization:

$$\max_{\alpha,\tau} \quad \Lambda \equiv \sum_j N^{y,j}\int\limits_{\omega_-}^{\omega_+} u(\omega(1-\tau)-s_j) + \delta u\left[\omega z_j(\omega) - \tfrac{\gamma(z_j)^2}{2} + \delta\left(\bar{P}_j(z_j) + \tfrac{\psi(1-z_j)^2}{2} + (1+r)s_j\right)\right] f(\omega)d\omega$$

subject to:

$$\bar{P}_j(\tau) = (1+n)\bar{\omega}\tau + \frac{\tau\theta}{\gamma-\beta_j\delta\psi}\left[(1-\tau\theta)E(\omega^2)+\beta_j\delta\alpha\tau\theta(E(\omega^2)-\psi)\right] \quad \text{for } j=n,s$$
$$\bar{P}_e(\tau) = (1+n)\bar{\omega}\tau + \frac{\tau\theta}{\gamma-\delta\psi}\left[(1-\tau\theta)E(\omega^2)+\delta\alpha\tau\theta(E(\omega^2)-\psi)\right]$$

The first order conditions for, respectively, α and τ, are:

$$\frac{\partial\Lambda}{\partial\tau} : -\sum_j N^{y,j}\int\limits_{\omega_-}^{\omega_+}\left[\omega u'(c_y^j) - \delta u'(c_o^j)\left(-\omega\theta z_j + \delta\frac{\partial\bar{P}_j(\tau)}{\partial\tau}\right)\right] = 0 \tag{20}$$
$$\frac{\partial\Lambda}{\partial\alpha} : -\sum_j N^{y,j}\int\limits_{\omega_-}^{\omega_+}\left[u'(c_o^j)\left(\frac{\partial z_j}{\partial\alpha}\left[\omega(1-\tau\theta)-z_j(\gamma-\psi\delta)-\psi\delta\right]+\delta\frac{\partial\bar{P}_j}{\partial\alpha}\right)\right] = 0 \tag{21}$$

where:

$$\frac{\partial\bar{P}_j}{\partial\alpha} = \frac{\beta_j\delta\left(\tau\theta\right)^2}{\gamma-\beta_j\delta\psi}\left[(E(\omega^2)-\psi)\right] \text{ for } j=s,n$$
$$\frac{\partial\bar{P}^e}{\partial\alpha} = \frac{\delta\left(\tau\theta\right)^2}{\gamma-\delta\psi}\left[(E(\omega^2)-\psi)\right]$$

Following Casamatta et al. (2005), the first FOC has a straightforward interpretation: if there are not liquidity constraints, standard results in linear income taxation apply: it is optimal to set $\tau^{opt} = 1$ in the first period, since young's labor supply is inelastic, and to redistribute through lump-sum transfers in the second period, setting, for example, $\alpha^{opt} = 0$, *i.e.* the pension includes only a Beveridgean component. However, with liquidity constraints, it is no longer true that it is optimal to impose a 100% income tax in the first period: consumption in young age must be positive. Therefore, optimality requires $\tau^{opt} < 1$.

Replacing the expressions for $\frac{\partial \bar{P}_j}{\partial \alpha}$ into (21) and rearranging, we get the expression for the optimal Bismarckian factor α^{opt}:

$$\alpha^{opt} = M \frac{\int_{\omega_-}^{\omega_+} u'(c_o)\tau\theta(E(\omega^2) - \psi) f(\omega)\, d\omega + \frac{N^{y,TI}}{\gamma - \beta\delta\psi} \int_{\omega_-}^{\omega_+} u'(c_o)\omega^2 (1-\tau\theta)(1-\beta+\omega) f(\omega)\, d\omega}{\delta\tau\theta\left(\frac{N^{y,e}}{\gamma-\delta\psi} E\left(u'(c_o)\omega^2\right) - \frac{N^{y,TI}}{\gamma-\beta\delta\psi} E\left(u'(c_o)\omega\right)\right)} \tag{22}$$

where $M = \frac{\tau\theta N(\gamma-\delta\psi)+\delta\psi N^{y,e}(1-\beta)}{(\gamma-\delta\psi)(\gamma-\beta\delta\psi)}$ is a constant term and $N^{y,TI} = N^{y,n} + N^{y,s}$ is the number of time inconsistent individuals. Expression (22) implies that the planner uses that the Bismarckian parameter α as a commitment device the helps time inconsistent workers to increase their welfare. To understand why, first notice that α^{opt} is an increasing function of $N^{y,TI}$: the higher is the number of time inconsistent individuals, the higher is the Bismarckian component of the pension formula. The intuition for this result is simple: from Lemma 1, we know that naive's retirement age is increasing in α. Since in the second best setting the planner can not force individuals to retire at the optimal z_j, establishing a tighter link between working career through a higher α^{opt} is a way to induce workers not to quit earlier their job and total welfare (Proposition 1)[22]. Moreover, the term M is increasing in overconfidence ($(1-\beta)$): the more naive overestimate their retirement age, ΔZ, and pension benefits, ΔP, the tighter the link between pension and retirement age should be.

8 The Political Equilibrium

In this section we study a political economy model in which all individuals vote over the payroll tax $\tau \in [0,1]$, and the Bismarckian factor, $\alpha \in [0,1]$. The political game works as follows: elections take place every period: each individual votes once in his life. Since each individual has zero mass, we assume that voting is sincere. Because of the bidimensionality of the voting space, a Condorcet winner may not exist. We handle this problem by using the notion of structure induced equilibrium defined by Shepsle (1979), which ensures the existence of an equilibrium if the multidimensional voting game is transformed into an issue-by-issue voting game. We first determine the majority voting equilibrium payroll tax, for a given level of redistribution: $\tau^{mv}(\alpha)$. Then, for every value of the tax rate, we compute the majority voting level of redistribution, for given payroll tax, $\alpha^{mv}(\tau)$. The (structure induced) equilibrium of our game (τ^s, α^s), if any, is the point at which these two functions intersect.

[22]Notice that, if all individuals were exponential $\left(N^{y,TI} = 0\right)$, the second term at the numerator and the second one at the denominator would disappear, and (22) would be lower.

8.1 Voting over τ

Because of A5, Preferred tax rates for young and old workers are the solutions to the following maximizations:

$$\max_{\tau\in[0,1]} V^y(\tau_j^y,\alpha;\hat{\beta}_j,\omega)$$
$$\max_{\tau\in[0,1]} V^o(\tau_j^o,\alpha;\beta_j,\omega)$$

where α is taken as given. Let us define, respectively, the indirect utility functions for young of type ($\hat{\beta}_j$,ω) and old of type (β_j,ω), as follows:

$$\begin{aligned} V^y(\tau_j^y,\alpha;\hat{\beta}_j,\omega) = {} & u(\omega(1-\tau_j^y)-\widehat{s_j})+ \\ & +\hat{\beta}_j\delta u\left(\omega\widehat{z_j}(1-\theta\tau_j^y)-\tfrac{\gamma(\widehat{z_j})^2}{2}+\delta\left[P(\widehat{z_j},\tau_j^y,\alpha)+\tfrac{\psi(1-\widehat{z_j})^2}{2}+(1+r)\widehat{s_j}\right]\right) \end{aligned} \quad (23)$$

$$V^o(\tau_j^o,\alpha;\beta_j,\omega) = u\left(\omega z_j(1-\theta\tau_j^o)-\tfrac{\gamma(z_j)^2}{2}+\beta_j\delta\left[P(z_j,\tau_j^o,\alpha)+\tfrac{\psi(1-z_j)^2}{2}+(1+r)s_j\right]\right) \quad (24)$$

8.1.1 The Young

Since the tax rate chosen by majority voting not only influences the rate of return of the PAYG system, but also the retirement age, it could be that, for some values of τ_j^y, an individual prefers the pension system to private saving and for some others not. The maximization problem should be modified as follows:

$$\max_{\widehat{s_j},\tau_j^y} u(\omega(1-\tau_j^y)-\widehat{s_j})+\hat{\beta}_j\delta u\left[\omega\widehat{z_j}(1-\tau\theta)-\frac{\gamma(\widehat{z_j})^2}{2}+\delta\left(P(\widehat{z_j},\tau_j^y,\alpha)+\frac{\psi(1-\widehat{z_j})^2}{2}\right)\right]$$

subject to:

$$\tau_j^y\geq 0,\qquad 0\leq\widehat{s_j}=c_y-\omega(1-\tau_j^y)\qquad j=e,n,s$$

The solution to the above maximization problem give us young's most preferred tax rate, $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$. In the appendix we show that results depend on the value of $\alpha\delta$. The following two propositions summarize our findings. We first consider the case $\alpha\delta<1/2$.

Proposition 3 *Suppose $\alpha\delta>1/2$. Then:*
(i) Preferred tax rates $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$ are positive for individuals with income $\omega_-\leq\omega<\tilde{\omega}$ and decreasing with ω;
(ii) There exists a threshold value $\omega'(<\tilde{\omega})$ such that saving are zero for individuals with $\omega\leq\omega'$, and positive and increasing with income thereafter;
(iii) For individuals with $\tau_y^+(\omega,\alpha;\hat{\beta}_j)>0$, there exists a threshold ω_b such that $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\hat{\beta}_j}<0$ for $\omega_-\leq\omega<\omega_b$ and $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\hat{\beta}_j}\geq 0$ for $\omega_b\leq\omega<\tilde{\omega}$;
(iv) The threshold $\tilde{\omega}$ is strictly decreasing in the degree of perceived time inconsistency $\hat{\beta}_j$: $\tilde{\omega}_{TI}>\tilde{\omega}_{TC}$;
(v) No young individual chooses a corner solution at $\tau_y^+(\omega,\alpha;\hat{\beta}_j)=1$;
(vi) It exists a threshold $\widehat{\omega}$ such that $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\alpha}>0$ for $\widehat{\omega}\leq\omega<\tilde{\omega}$, and $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\alpha}<0$ otherwise.

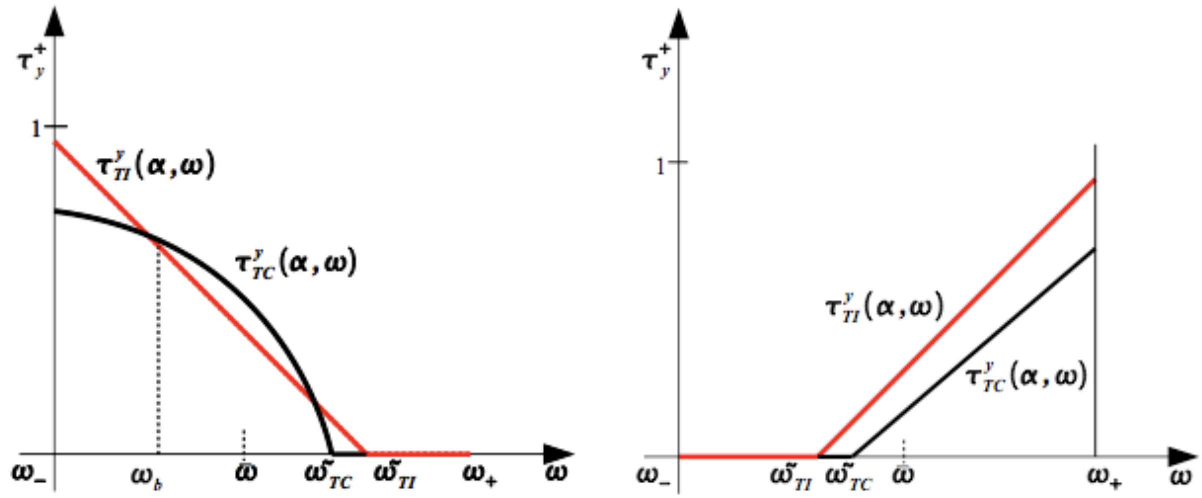

Figure 4: Preferred τ for time consistent and hyperbolic young: cases $\alpha\delta < 1/2$ (left) and $\alpha\delta > 1/2$ (right).

Proof. See Appendix B. ■

We now give a sketch of the main intuitions, with the help of Figure 4, where we denote with $\tau_{TC}^y \equiv \tau_y^+(\omega, \alpha; \hat{\beta}_j = 1)$ the preferred tax rates for *perceived* time consistent (naive and exponential) young, and with $\tau_{TI}^y \equiv \tau_y^+(\omega, \alpha; \hat{\beta}_j = \beta)$ the preferred tax for sophisticated.

In part *(i)* we show that, for given $\hat{\beta}$, only individuals with productivity levels up to $\tilde{\omega}$ support the social security system. Rich individuals, on the other hand, finance post retirement consumption only through private saving, and vote for $\tau = 0$. Moreover, $\tau_y^+(\omega, \alpha; \hat{\beta}_j)$ is decreasing with productivity, a result in line with the literature. Several effects determine this result: first, notice that current consumption, c_y^j, is increasing with ω. Consumption in second and third periods, c_o^j, is also increasing with income, provided that it is a normal good. Consequently, rich individuals would like to transfer more resource in the second period through a higher τ (income effect). Second, a substitution effect operates: if ω rises, the relative price of first and second period consumption decreases, and this effect is more pronounced for low income workers. For utility functions such that $\epsilon < 1$, as we assume, this substitution effect dominates income effect: low productivity individuals prefer larger τ than high productivity ones. Finally, assuming endogenous retirement age leads to a third effect (Casamatta et al., 2005): second period consumption increases with productivity, given that more productive individuals retire later (Lemma 1), and young rich raise their first period consumption by reducing the payroll tax.

Result *(ii)* is in line with Casamatta et al. (1999): it exists a threshold ω' such that individuals with productivity below ω' do not save and rely only on the pension system to finance third period consumption. On the other and, for productivities above ω', saving are positive and increasing with ω. Combining this result with *(i)*, we see that, for $\omega' \leq \omega \leq \tilde{\omega}$, we have interior solutions for both τ and

$\widehat{s_j}$. Furthermore, since saving are increasing with income and preferred tax rate are decreasing with it, we see that workers progressively replace the pension system with private savings, up to the threshold $\tilde{\omega}$, where preferred τ is zero.

Parts *(iii)* and *(iv)* are a novelty of this work follows from our behavioral assumption. In *(iii)* we compare, for given income level, sophisticated preferred tax rates, τ^y_{TI}, with those of exponential and naive, τ^y_{TC}. We show that it exists a *non-monotonic* relationship between the two tax rates: in particular, we have that $\tau^y_{TC} < \tau^y_{TI}$ for $\omega_- \leq \omega \leq \omega_b$, and $\tau^y_{TC} > \tau^y_{TI}$ thereafter. The intuition for the first part of the proposition is simple: notice that, for $\omega \leq \omega_b$, saving are zero and $z_s < z_e$, for a given ω. Sophisticated find optimal to set τ higher than exponential: a higher tax a second order loss, since it further reduces retirement age, but has first order positive effect on total utility, through a more generous pension system. Since the former effect is less marked for sophisticated, given that z_s is suboptimal, $\tau^y_{TC} < \tau^y_{TI}$. The second part of *(iii)* is more counterintuitive: we show that for $\omega > \omega_b$, $\tau^y_{TC} > \tau^y_{TI}$, *i.e.* sophisticated prefer a smaller payroll tax than exponential with the same ω. The intuition for this result comes from the "commitment device" argument, typical in the Economics and Psychology literature (Gruber and Köszegi, 2004). In absence of a publicly-provided commitment devices that would help hyperbolic individuals to overcome their self-control issues, sophisticated look for a "personal" commitment device that increases at the same time z_s and s_s, whose values are suboptimal. In Lemma 1 and Appendix B, we have shown that both retirement age and saving are decreasing with τ. Moreover, from Propositions 1 and 2, we know that increasing z_s and s_s towards z_e and s_e (normative retirement age and saving), increases sophisticated welfare as well. Therefore, the commitment device for hyperbolic individuals takes the form of a *lower* payroll tax.

In *(iv)*, we demonstrate that $\tilde{\omega}$, the income threshold such that private savings are preferred to the pension system, is greater for sophisticated than for exponential and naive: $\tilde{\omega}_{TI} > \tilde{\omega}_{TC}$. This is intuitive: whereas time consistent workers with $\omega \geq \omega\tilde{_{TC}}$ find unattractive the pension sytem, since they are able to transfer autonomously income in the second and third period, sophisticated with productivity levels $\omega \in [\tilde{\omega}_{TC}, \tilde{\omega}_{TI}]$, still find attractive it.

Part *(v)* is straightforward: with $\tau^+_y(\omega, \alpha; \hat{\beta}_j) = 1$, marginal utility of consumption in the first period tends to infinite, and thus young prefers tax smaller than 1.

In *(vi)*, we show that most preferred tax rates are increasing in α. The intuition for this result is the following: suppose that α increases; for rich individuals (those with $\omega \geq \widehat{\omega}$) this is beneficial: it increases $\hat{z}_j$, $P(\hat{z}_j)$ and c^j_o. Moreover, making the system more Bismarckian is also beneficial. Therefore, to benefit more from these changes, preferred τ must increase too. However, for poor individuals, $\tau^+_y(\omega, \alpha; \hat{\beta}_j)$ is decreasing with α: when $\alpha\delta > 1/2$, *i.e.* redistribution is already low, rising α further increases the Bismarckian part. Poor individuals have to rise their retirement age to have resources in period 3: this can be done only through a lower payroll tax. It follows that τ must be decreasing with α, in order to increase both labor supply and $P(\hat{z}_j)$. On top of that, by continuity of the preferred tax rates function, there exists a threshold level of α, $\hat{\alpha} \equiv \left[\frac{\left[\left(\frac{\bar{\omega}}{\omega}\right)-(1+n)\delta\right]\frac{\gamma-\hat{\beta}_j\delta\psi}{\delta\theta}-\omega(1-\theta\tau(2+\hat{\beta}_j)-\hat{\beta}_j\delta\psi)}{4\omega\theta\tau\delta\hat{\beta}_j} \right]$ such that $\tau_y^+(\omega, \alpha; \hat{\beta}_j)$ is increasing in α for $\alpha \geq \hat{\alpha}$ and decreasing otherwise.

We move now to the second case, $\alpha\delta < 1/2$, which implies that the redistributive part in the pension formula is high.

Proposition 4 *Suppose $\alpha\delta < 1/2$. Then:*

(i) Preferred tax rates $\tau_y^+(\omega, \alpha; \hat{\beta}_j)$ are positive and increasing with productivity for individuals with $\tilde{\omega} < \omega \leq \omega^+$;

(ii) The threshold $\tilde{\omega}$ is strictly increasing in the degree of perceived time inconsistency $\hat{\beta}_j$;

(iii)For individuals with $\tau_y^+(\omega, \alpha; \hat{\beta}_j) > 0$, the preferred tax rate is decreasing in the degree of perceived time consistency $\hat{\beta}_j$, $\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \hat{\beta}_j} < 0$.

(iv) No young individual chooses a corner solution at $\tau_y^+(\omega, \alpha; \hat{\beta}_j) = 1$.

(v) For individuals above the average income, i.e. $(\bar{\omega} \leq) \tilde{\omega} \leq \omega < \omega^+$, we have that $\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \alpha} > 0$.

Proof. See Appendix B. ■

In part *(i)* we show that only rich individuals prefer a positive tax rate: by increasing τ, they can transfer more resources from first to second period, provided that consumption is a normal good. Poor workers, on the other side, will retire earlier and prefer to keep the tax rate as low as possible, in order to increase retirement age and saving.

Part *(ii)* says that the higher is the perceived time consistency of the worker, the less support the social security system receives: for sophisticated, the effect described in part *(i)* emerges for lower productivity levels, because these agents realize they are not able to respect their plans.

In part *(iii)*, we show that time-inconsistent agents prefer a higher tax rate than exponential, for a given productivity level. The intuition goes as follows: given that α is low, rich workers increase $P(\hat{z}_j)$ through a higher tax rate, and this effect is more marked for hyperbolic individuals. The commitment effect of proposition 3 does not operate here: those in favor of a positive τ have already positive saving and high retirement age. The increase of utility to a lower τ (which increases z and s) has a second order effect than the increase in utility due to a more generous pension, given that $\partial z/\partial\tau$ is decreasing with income. Time consistent individuals, on the other hand, are less interested in transferring resources through the PAYG system, given their optimal choices, thus preferring a lower, although positive, τ.

Part *(v)* is similar to Proposition 3: here, given that $\tilde{\omega} \geq \bar{\omega}$, only the case $\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \alpha} > 0$ is relevant.

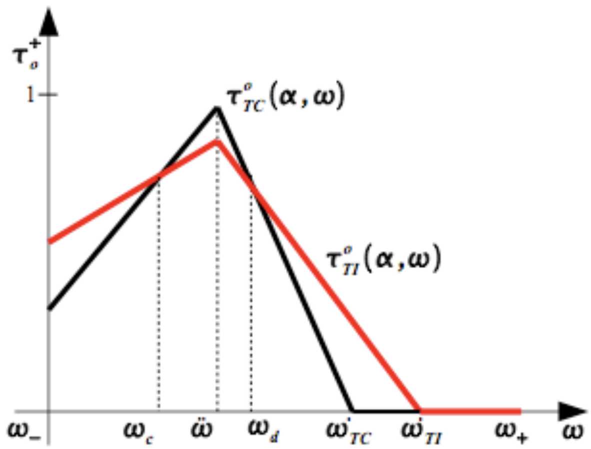

Figure 5: Preferred τ for time consistent and hyperbolic old.

8.1.2 The Old

Maximization of (24) leads to the following proposition.

Proposition 5 *Preferred tax rates for old have the following properties:*

(i) Preferred tax rates $\tau_o^+(\omega,\alpha;\beta_j)$ *are positive for individuals with productivity* $\omega_- \leq \omega \leq \dot{\omega}$ *and always lower than 1.*

(ii) Preferred tax rates $\tau_o^+(\omega,\alpha;\beta_j)$ *are increasing for productivity levels* $\omega_- \leq \omega < \ddot{\omega}$ *and decreasing in* ω *for* $\ddot{\omega} \leq \omega \leq \dot{\omega}$*;*

(iii) The threshold $\dot{\omega}$ *is strictly decreasing in* β_j*;*

(iv) Preferred tax rates $\tau_o^+(\omega,\alpha;\beta_j)$ *are increasing with* β_j *for income levels* $\omega_c < \omega < \omega_d$ *and decreasing with* β_j*,* $\forall\omega$ *such that* $\omega_- \leq \omega \leq \omega_c$ *and* $\omega_d \leq \omega \leq \dot{\omega}$*;*

(v) For individuals above the average income, i.e. $\bar{\omega} \leq \omega < \tilde{\omega}$*, we have that* $\frac{\partial\tau_o^+(\omega,\alpha;\beta_j)}{\partial\alpha} > 0$*, whereas for very poor individuals,* $\frac{\partial\tau_o^+(\omega,\alpha;\beta_j)}{\partial\alpha} < 0$.

Proof. See Appendix B. ■

We provide now a sketch of the main intuitions, with the help of Figure 5, where $\tau_{TC}^o \equiv \tau_o^+(\omega,\alpha;\beta=1)$ denotes preferred tax rates of time consistent old and $\tau_{TI}^o \equiv \tau_o^+(\omega,\alpha;\beta)$ those of naive and sophisticated.

In part *(i)* we demostrate that only by agents with productivity levels below $\dot{\omega}$ support the social security system; this result contrasts with the classical literature on positive social security (Casamatta et al. 1999), where every old votes for a positive τ and at least a fraction of them in favor of $\tau = 1$. This result follows from our assumptions of elastic labor supply in period 2 and the concavity of the perceived pension function: increasing of τ has two opposite effects on individuals' utility: it reduces retirement age

(Lemma 1), but it increases pension benefits, provided that the tax rate is below the threshold $\hat{\tau}$ (Lemma 2)). For individuals with productivity up to $\dot{\omega}$, the second effect overweights the first one: preferred tax rates are positive.

In *(ii)* we show that, for a given β, $\tau_o^+(\omega, \alpha; \beta_j)$ is a concave function with a maximum at $\ddot{\omega}$. This effect is due to the two effects pointed out in part *(i)* and a third effect: in Lemma 1, we show that $\frac{\partial \hat{z}_j}{\partial \tau}$ is decreasing with ω, *i.e.* poor reduces more than rich z in response to an increase in the payroll tax. It follows that, for low productivity levels, this third effect prevails, and preferred tax rate are low. As income increases, the reduction of z is more than compensated by the increase of P, and τ increases with income. For income levels $\omega \in [\ddot{\omega}, \dot{\omega}]$ saving represent a substitute for teh PAYG system: preferred τ is decreasing with income, up to $\dot{\omega}$, where old exclusively rely on private saving and vote for $\tau_o^+ = 0$.

In *(iii)* we claim that exponential old are less likely to support the pension system. The threshold such that private saving are preferred to the social security system is lower for time consistent old: $\dot{\omega}_{TI} > \dot{\omega}_{TC}$. Rich exponential have optimal private saving and a longer career, and a generous pension system is not necessary. On the other hand, old naive and sophisticated, who realize the suboptimality of their choices, support social security for higher productivity levels in order to raise post-retirement consumption.

In *(iv)*, we show that, for given ω, preferred tax rates are non-monotonic in β_j; more precisely, for productivity levels below ω_c and above ω_d, time inconsistent individuals prefer higher τ than exponential. Both hyperbolic poor (those with $\omega_- \leq \omega \leq \omega_c$) and hyperbolic rich (those with $\omega_d \leq \omega \leq \dot{\omega}$) prefer a more generous system in order to increase post retirement consumption, but the reasons behind such behavior differ: since α is given, the former group would like to augment the size of Beveridgean component of $P(z_j)$, whereas the latter the size of the Bismarckian part. Hyperbolic agents, *i.e.* those with $\omega_c < \omega < \omega_d$, on the other hand, prefer a lower τ than exponential. The intuition goes as follows: for given α, these individuals are less interested in increasing the size of the Beveridgean part, since their productivity level is around the average; moreover, their lack of self control makes also the Bismarckian part less attractive. It follows that a commitment device strategy operates here: lowering τ increases consumption for second and third periods, increases z, but has a cost, namely the pension benefits are reduced. However, since period three utility is discounted at the hyperbolic factor $\beta\delta$, this loss has only a second order effect.

Intuition for *(v)* goes exactly as for Proposition 3, and thus is omitted.

8.2 Equilibrium Tax Rate

In the appendix, we prove that preferences over τ are single-peaked, both for young and old voters, and the median voter theorem applies. In the following, to highlight our contribution with respect to traditional political economy model of social security, we compare the equilibrium tax rate of our hyperbolic voting game, $\tau_{TI}^{mv}(\alpha)$, to the tax rate that would emerge in a model with exponential discounting, $\tau_{TC}^{mv}(\alpha)$.

Since preferred tax rates depends crucially on the value of the parameter $\alpha\delta$, we will consider separately the two cases.

Case $\alpha\delta > 1/2$

Figure 6 and Proposition 6 illustrate the Condorcet winner of the voting game[23].

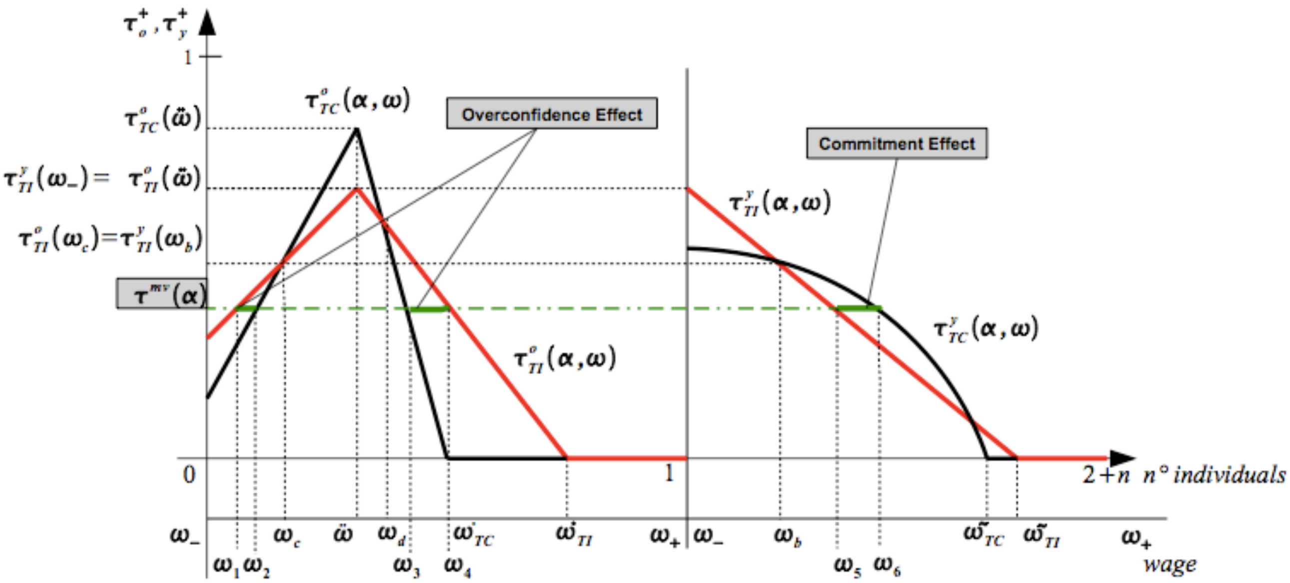

Figure 6: Equilibrium τ when $\alpha\delta > 1/2$ and $\tau^o_{TC}(\ddot{\omega}) \geq \tau^y_{TI}(\omega_-)$.

Proposition 6 *If $\alpha\delta \geq 1/2$ and $\tau^o_{TC}(\ddot{\omega}) \geq \tau^y_{TI}(\omega_-)$, the majority voting equilibrium tax rate $\tau^{mv}(\alpha)$ satisfies the following conditions:*

(i) If $(1+n)N^s \int_{\tilde{\omega}_{TI}}^{\omega_+} f(\omega)d\omega + (N^s + N^n)\int_{\dot{\omega}_{TI}}^{\omega_+} f(\omega)d\omega > N(2+n)/2$, then $\tau^{mv}_{TI}(\alpha) = 0$.

(ii) The majority voting equilibrium $\tau^{mv}_{TI}(\alpha)$ is positive if and only if:

$$N^o \int_{\omega_-}^{\dot{\omega}_{TC}} f(\omega)d\omega + (1+n)N^y \int_{\omega_-}^{\tilde{\omega}_{TC}} f(\omega)d\omega + \underbrace{(N^s + N^n)\int_{\dot{\omega}_{TC}}^{\dot{\omega}_{TI}} f(\omega)d\omega + (1+n)N^s \int_{\tilde{\omega}_{TC}}^{\tilde{\omega}_{TI}} f(\omega)d\omega}_{\text{extra support for SS}} \geq \frac{N(2+n)}{2}$$

(iii) $\tau^{mv}_{TI}(\alpha)$ is the rate preferred by the workers with earnings $\omega_1, \omega_2, \omega_3, \omega_4,$ ω_5 or ω_6 such that:

[23] In the appendix we show that $\tau^o_{TC}(\ddot{\omega})$, the maximal preferred tax rate for time consistent old, and $\tau^y_{TI}(\omega_-)$, the maximal preferred tax rate for hyperbolic young are not comparable: both cases are possible. However, the case $\tau^o_{TC}(\ddot{\omega}) < \tau^y_{TI}(\omega_-)$ gives the same results of the case $\tau^o_{TC}(\ddot{\omega}) \geq \tau^y_{TI}(\omega_-)$. Therefore, without loss of generality, we can focus only on the second one.

$$N^o\left(\int_{\omega_2}^{\omega_3} f(\omega)d\omega + (1+n)\int_{\omega_-}^{\omega_6} f(\omega)d\omega\right) + \underbrace{(N^s+N^e)\left(\int_{\omega_1}^{\omega_2} f(\omega)d\omega + \int_{\omega_3}^{\omega_4} f(\omega)d\omega\right)}_{\textit{Overconfidence Effect (+)}} +$$

$$\underbrace{-\,N^s(1+n)\int_{\omega_5}^{\omega_6} f(\omega)d\omega}_{\textit{Commitment Effect (-)}} = \frac{N(2+n)}{2}$$

and

$$\tau_{TI}^{mv}(\alpha) = \tau_o^+(\omega_1,\alpha;\beta_j) = \tau_o^+(\omega_2,\alpha;\beta_j) = \tau_o^+(\omega_3,\alpha;\beta_j) = \tau_o^+(\omega_4,\alpha;\beta_j) = \tau_y^+(\omega_5,\alpha;\hat{\beta}_j) = \tau_y^+(\omega_6,\alpha;\hat{\beta}_j)$$

(iv) $\tau_{TI}^{mv}(\alpha) < \tau_{TC}^{mv}(\alpha)$ *if and only if:*

$$(N^s+N^e)\left(\int_{\omega_1}^{\omega_2} f(\omega)d\omega + \int_{\omega_3}^{\omega_4} f(\omega)d\omega\right) < N^s(1+n)\int_{\omega_5}^{\omega_6} f(\omega)d\omega$$

We sketch the proof, which relies on standard arguments, through Figure 6, where we have supposed that the median income is such that the equilibrium payroll tax is $\tau_{TI}^{mv}(\alpha)$.

In *(i)*, we show that social security can not be sustained as an equilibriumt, if the number of voters in favor of a positive tax rate does not represent half of the population.

Part *(ii)* follows directly from Propositions 3 *(iii)* and 5 *(iii)*; we show that, for a given productivity level, time inconsistent agents are more favorable to support a pension system: the thresholds $\tilde{\omega}$ and $\dot{\omega}$, i.e. tax rates such that preferred τ are positive, are both decreasing with $\hat{\beta}_j$[24]. If there are enough hyperbolic individuals with productivity $\omega \in [\dot{\omega}_{TC}, \dot{\omega}_{TI}]$ and $\omega \in [\tilde{\omega}_{TC}, \tilde{\omega}_{TI}]$, the pension system is more likely to be supported as an equilibrium. The *extra-support* for social security due to time inconsistency is given by the term $(N^{o,s}+N^{o,n})\int_{\dot{\omega}_{TC}}^{\dot{\omega}_{TI}} f(\omega)d\omega + (1+n)N^{o,s}\int_{\tilde{\omega}_{TC}}^{\tilde{\omega}_{TI}} f(\omega)d\omega$.

In part *(iii)* we show that the majority voting equilibrium tax rate $\tau_{TI}^{mv}(\alpha)$ is determined by two opposite forces. The first one, the **overconfidence** effect, increases $\tau_{TI}^{mv}(\alpha)$: old naive have realized the suboptimality of their choices, and favor a more generous system in order to compensate the drop in post retirement consumption. The second one, the **commitment** effect, goes in the opposite direction: for a given productivity level, a sophisticated young favor a lower τ than a young exponential. As stressed above, sophisticated young, aware of their present bias, see a lower payroll tax as a commitment device that increases both z_s and s_s and mitigate the negative effects of time inconsistency.

In part *(iv)* of the proposition we compare the hyperbolic equilibrium tax rate, $\tau_{TI}^{mv}(\alpha)$ to the one of a standard exponential model, $\tau_{TC}^{mv}(\alpha)$. If the coalition made of hyperbolic young and rich individuals outnumbers those made of poor old and time consistent young, i.e. the commitment is higher than the overconfidence, we have, in the hyperbolic equilibrium, a smaller, pension system than in the exponential equilibrium: $\tau_{TI}^{mv}(\alpha) < \tau_{TC}^{mv}(\alpha)$.

[24]Most preferred tax rates for exponential old with productivity between $\dot{\omega}_{TC}$ and $\dot{\omega}_{TI}$ and for exponential and naive young with productivity between $\tilde{\omega}_{TC}$ and $\tilde{\omega}_{TI}$ are zero, while for sophisticated young and naive old, these tax rates are positive.

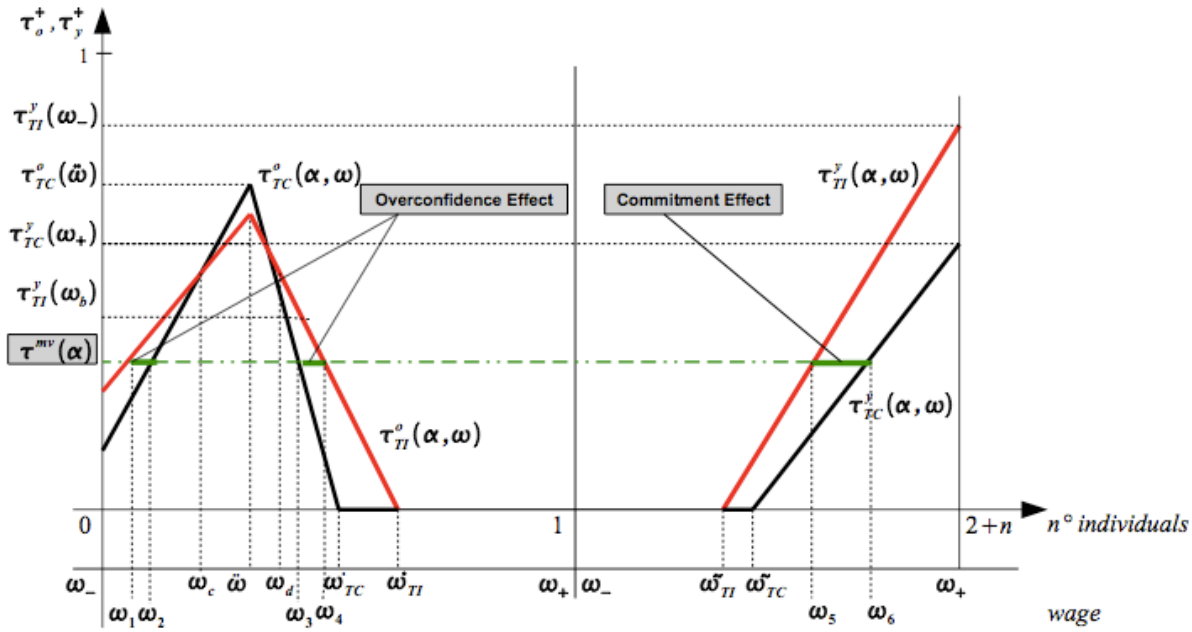

Figure 7: Equilibrium τ when $\alpha\delta < 1/2$.

Case $\alpha\delta < 1/2$

When the Bismarckian part of the pension system is below $1/2\delta$, the characterization of the equilibrium is slightly modified. Proposition 7 summarizes our findings.

Proposition 7 *If $\alpha\delta < 1/2$, the majority voting equilibrium tax rate satisfies the following conditions:*
(i) If $(1+n)N^s \int_{\omega_-}^{\tilde{\omega}_{TI}} f(\omega)d\omega + (N^s+N^n)\int_{\dot{\omega}_{TI}}^{\omega_+} f(\omega)d\omega > N(2+n)/2$, then the majority voting equilibrium tax rate $\tau_{TI}^{mv}(\alpha)$ is 0.

(ii) The majority voting equilibrium $\tau_{TI}^{mv}(\alpha)$ is positive if:

$$N^o \int_{\omega_-}^{\dot{\omega}_{TC}} f(\omega)d\omega + (1+n)N^y \int_{\tilde{\omega}_{TC}}^{\omega_+} f(\omega)d\omega + \underbrace{(N^s+N^n)\int_{\dot{\omega}_{TC}}^{\dot{\omega}_{TI}} f(\omega)d\omega + (1+n)N^s \int_{\tilde{\omega}_{TC}}^{\tilde{\omega}_{TI}} f(\omega)d\omega}_{extra\ support\ for\ SS} \geq \frac{N(2+n)}{2}$$

(iii) The majority voting equilibrium tax rate is the rate preferred by the workers with earnings $\omega_1, \omega_2, \omega_3, \omega_4$, ω_5 or ω_6 such that:

$$N^o\left(\int_{\omega_2}^{\omega_3} f(\omega)d\omega + (1+n)\int_{\omega_6}^{\omega_+} f(\omega)d\omega\right) + \underbrace{(N^s+N^e)\left(\int_{\omega_1}^{\omega_2} f(\omega)d\omega + \int_{\omega_3}^{\omega_4} f(\omega)d\omega\right)}_{Overconfidence\ Effect\ (+)}$$

$$+ \underbrace{N^s(1+n)\int_{\omega_5}^{\omega_6} f(\omega)d\omega}_{Commitment\ Effect\ (+)} = \frac{N(2+n)}{2}$$

and

$$\tau_{TI}^{mv}(\alpha) = \tau_o^+(\omega_1, \alpha; \beta_j) = \tau_o^+(\omega_2, \alpha; \beta_j) = \tau_o^+(\omega_3, \alpha; \beta_j) = \tau_o^+(\omega_4, \alpha; \beta_j) = \tau_y^+(\omega_5, \alpha; \hat{\beta}_j) = \tau_y^+(\omega_6, \alpha; \hat{\beta}_j)$$

(iv) If $N^{y,TI} \neq 0$ *(or* $N^{o,TI} \neq 0$*),* $\tau_{TI}^{mv}(\alpha) > \tau_{TC}^{mv}(\alpha)$*.*

Parts *(i)* and *(ii)* parallel Proposition 6. The only difference is given by *(iii)*, in which we determine the equilibrium tax rate, $\tau_{TI}^{mv}(\alpha)$. Contrary to the previous case, here both the overconfidence and the commitment effect go in the same direction, and contributes to increase the equilibrium payroll tax. The intuition for this result is the following: now the commitment device for sophisticated young is a *higher* τ; since redistribution is high, and only rich favor a positive tax, there is no need to further increase z_j. The only way to transfer resources in the second period is to increase the size of the system: the loss in utility due to a reduced retirement age is more the compensated by the increase in the generosity of the system. It follows that, when $\alpha\delta < 1/2$, the equilibrium payroll tax $\tau_{TI}^{mv}(\alpha)$ is higher than $\tau_{TC}^{mv}(\alpha)$: the winning coalition is now composed by time inconsistent agents (old and young), who prefer a higher tax than time consistent individuals.

8.3 Voting over the Bismarckian Factor

In the appendix, we show that preferences over α are single-crossing both for young and for old: hence, a voting equilibrium on α exists. When τ is kept fixed, most preferred levels of α are the solution to the following problems:

$$\max_{\alpha \in [0,1]} V^o(\tau, \alpha; \beta_j, \omega) \tag{25}$$

$$\max_{\alpha \in [0,1]} V^y(\tau, \alpha; \hat{\beta}_j, \omega) \tag{26}$$

where the expressions for the two indirect utility functions are given, respectively, by equations (23) and (24). Because of A5, the problem is not same for the 2 generations: young evaluates future utility and consumption according to the perceived discount factor $\hat{\beta}_j \delta^2$, whereas old uses the true discount factor $\beta_j \delta$.

8.3.1 The Old

The following proposition summarizes our findings about old's preferred levels of the Bismarckian parameter, $\alpha_o^+(\tau, \omega; \beta_j)$ (see Figure 8 for a graphical interpretation).

Proposition 8 *Old's preferred levels of* α *have the following properties:*

(i) It exists a productivity level ω_e *such that* $\alpha_o^+(\tau, \omega; \beta_j) > 0$ *for* $\omega_e \leq \omega \leq \omega_+$*. Otherwise,* $\alpha_o^+(\tau, \omega; \beta_j) = 0$*; moreover* $\frac{\partial \alpha_o^+(\tau, \omega; \hat{\beta}_j)}{\partial \omega} > 0$*;*

(ii) The threshold ω_e *is decreasing in the degree of true time inconsistency* $\beta_j : \omega_e^{TC} < \omega_e^{TI}$*;*

(iii) $\frac{\partial \alpha_o^+(\tau,\omega;\beta_j)}{\partial \beta_j} > 0$*;*

(iv) For individuals above ω_e*, we have that* $\frac{\partial \alpha_o^+(\tau,\omega;\beta_j)}{\partial \tau} > 0$.

Proof. See Appendix B. ■

In part *(i)*, we show that, for given β, only old with $\omega > \omega_e$ favor $\alpha_o^+ > 0$, whereas for individuals with $\omega \leq \omega_e (< \bar{\omega})$, $\alpha_o^+ = 0$. To see why poor prefer a purely Beveridgean system, we have to consider how changes in the Bismarckian factor influence individuals' utility: first, higher α increases z, as it establishes a closer link between working career and pension benefits; second, higher α reduces redistribution. For productivities below ω_e, the second effect prevails, whereas for $\omega > \omega_e$ agents prefer to decrease redistribution and to make the system more Bismarckian. Moreover, as z is increasing with ω, also α is increasing with productivity.

Part *(ii)* shows that, for given ω, the income threshold such that $\alpha_o^+(\tau,\omega;\beta_j) > 0$ is lower for exponential than for hyperbolic: time consistent individuals, who have a longer career, prefer to have a more Bismarckian system than sophisticated with the same income level. An interesting corollary is that also individuals with productivity below the average prefer a positive α: for those with $\omega \in [\omega_e, \bar{\omega}]$, the increase in utility due to a longer career and a Bismarckian pension more than compensate the loss due to a reduction of the flat part of the transfer.

It follows that (part *iii*) that hyperbolic old prefer a lower α_o^+ than exponential with the ω; intuitively, although their income level should make them in favor of a Bismarckian system, their time inconsistency, that leads to lower z and c_o, makes them in favor of a smaller link between length of the working career and $P(z)$.

In *(iii)*, we show that, for $\omega \geq \omega_e$, $\alpha_o^+(\tau,\omega;\beta_j)$ is increasing with τ: augmenting the payroll tax reduces z_j and, therefore, only individuals with productivity above ω_e would like to increase the weight attached to the Bismarckian part, as to increase both their z_j and their $P(z_j)$.

8.3.2 The Young

The following proposition summarizes our findings about $\alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right)$, the most preferred level of the weight of the Bismarckian part in the benefit formula for young workers.

Proposition 9 *For young, the most preferred level of* α *has the following properties:*

(i) There exist a threshold income level ω_f *such that* $\alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right) = 0$ *for* $\omega_- \leq \omega \leq \omega_f$ *and* $\alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right) > 0$ *otherwise; moreover, when* $\alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right) > 0$*, we have* $\frac{\partial \alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right)}{\partial \omega} > 0$*;*

(ii) The threshold ω_f *is decreasing in the degree of true time inconsistency* $\beta_j : \omega_f^{TC} < \omega_f^{TI}$*;*

(iii) $\frac{\partial \alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right)}{\partial \hat{\beta}_j} > 0$*;*

(iv) For $\omega \geq \omega_f$*, we have that* $\frac{\partial \alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right)}{\partial \tau} > 0$

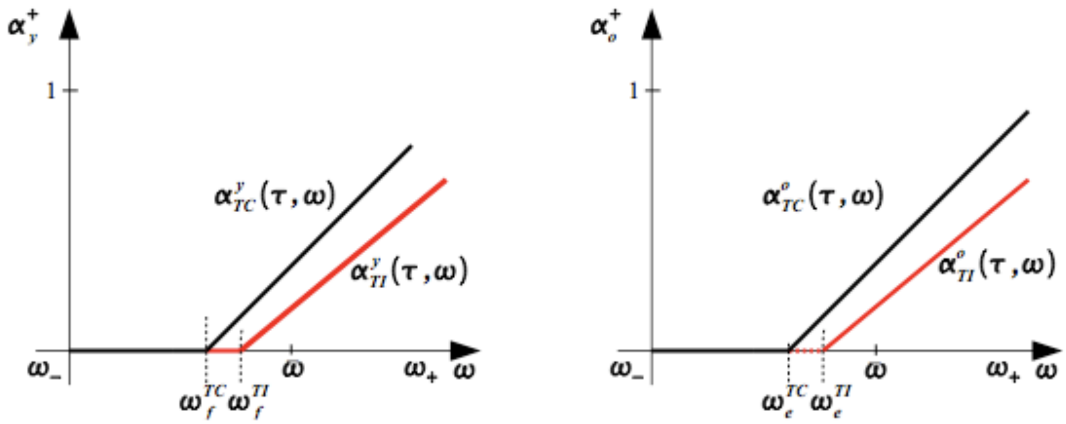

Figure 8: Preferred α for time consistent and hyperbolic old (left) and young (right).

Proof. See Appendix B. ■

Intuitions are similar to those behind Proposition 8 and are omitted.

8.4 Equilibrium Bismarckian Factor

Combining our results about $\alpha_y^+\left(\tau,\omega;\hat{\beta}_j\right)$ and $\alpha_o^+\left(\tau,\omega;\beta_j\right)$, we determine the equilibrium value of the Bismarckian parameter, depicted also in Figure 9.

Proposition 10 *The equilibrium $\alpha_{TI}^{mv}(\tau)$ satisfies the following conditions:*

(i)If $N^o\int_{\omega_-}^{\omega_e^{TC}} f(\omega)d\omega + (N^s+N^n)\int_{\omega_e^{TC}}^{\omega_e^{TI}} f(\omega)d\omega + (1+n)N^y\int_{\omega_-}^{\omega_f^{TC}} f(\omega)d\omega + (N^s+N^n)\int_{\omega_f^{TC}}^{\omega_f^{TI}} f(\omega)d\omega >$ $N(2+n)/2$, then $\alpha_{TI}^{mv}(\tau)=0$.

(ii) The majority voting equilibrium $\alpha_{TI}^{mv}(\tau)$ is positive if:

$$N^o\int_{\omega_e^{TC}}^{\omega_+} f(\omega)d\omega+(1+n)N^y\int_{\omega_f^{TC}}^{\omega_+} f(\omega)d\omega-\underbrace{\left[(N^s+N^n)\int_{\omega_e^{TC}}^{\omega_e^{TI}} f(\omega)d\omega+(1+n)N^s\int_{\omega_f^{TC}}^{\omega_f^{TI}} f(\omega)d\omega\right]}_{\textit{extra support for redistribution}}\geq\frac{N(2+n)}{2}$$

(iii) The majority voting equilibrium tax rate is the rate preferred by the workers with earnings $\omega_1,\omega_2,\omega_3,\omega_4$ such that:

$$N^o\left(\int_{\omega_1}^{\omega_+} f(\omega)d\omega+(1+n)\int_{\omega_3}^{\omega_+} f(\omega)d\omega\right)-\underbrace{(N^s+N^e)\left(\int_{\omega_1}^{\omega_2} f(\omega)d\omega\right)}_{\textit{Overconfidence Effect (-)}}-\underbrace{N^s(1+n)\int_{\omega_3}^{\omega_4} f(\omega)d\omega}_{\textit{Resignation Effect (-)}}=\frac{N(2+n)}{2}$$

and

$$\alpha^{mv}(\tau)=\alpha_o^+(\omega_1,\tau;\beta_j)=\alpha_o^+(\omega_2,\tau;\beta_j)=\alpha_y^+(\omega_3,\tau;\hat{\beta}_j)=\alpha_y^+(\omega_4,\tau;\hat{\beta}_j)$$

(iv) If $N^{y,TI}\neq 0$ (or $N^{o,TI}\neq 0$), $\alpha_{TI}^{mv}(\tau)<\alpha^{opt}$.

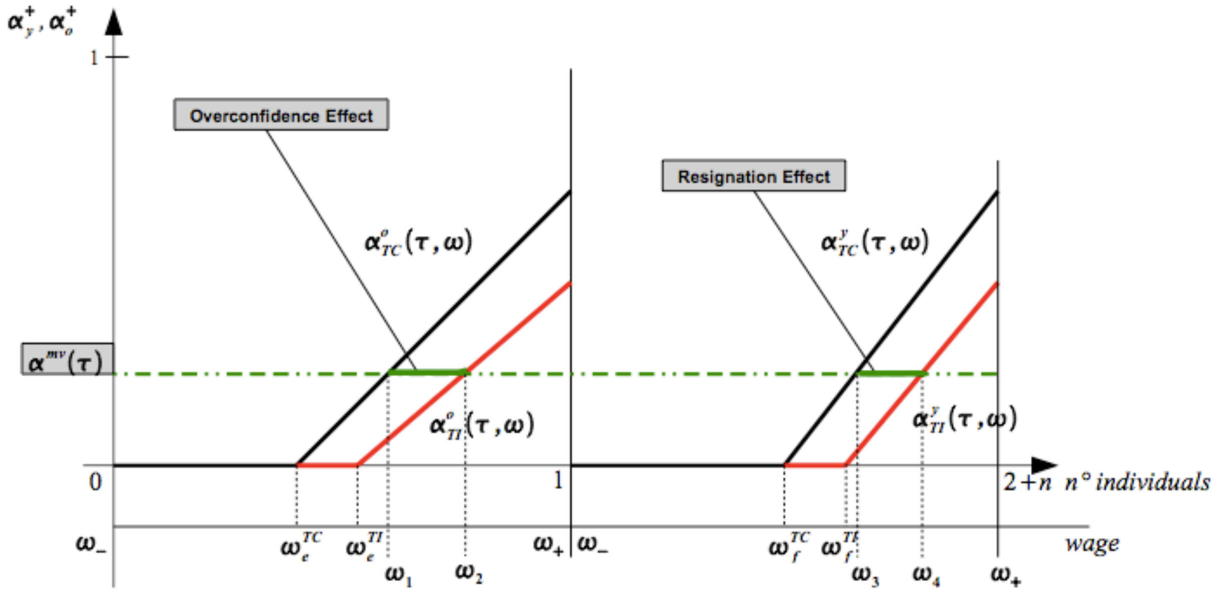

Figure 9: The majority voting equilibrium α, $\alpha_{TI}^{mv}(\alpha)$.

Parts *(i)* and *(ii)* of the proposition are intuitive: if the number of poor and time inconsistent is high enough, the equilibrium pension system is purely Beveridgean. In particular, the extra support for $\alpha_{TI}^{mv}(\tau) = 0$ is given by middle-income hyperbolic individuals, *i.e.* those with productivity levels $\omega \in [\omega_e^{TC}, \omega_e^{TI}]$ and $\omega \in [\omega_f^{TC}, \omega_f^{TI}]$.

In *(iii)*, we determine the majority voting equilibrium level of $\alpha_{TI}^{mv}(\tau)$: as for the payroll tax, time inconsistency introduces two effects in the political game: the **overconfidence** effect, displayed by old, is due to the fact that hyperbolic try to compensate the loss due to impatience by increasing the redistributive part of the pension formula, *i.e.* lowering α. Also the **resignation** effect lowers the equilibrium α: sophisticated young already know that they will not be able to stick with their optimal plans, and therefore support a more redistributive pension compared to time consistent individuals with the same ω.

In part *(iv)*, to stress the differences between our behavioral model and a standard model with exponential preferences, we distinguish between $\alpha_{TI}^{mv}(\tau)$, the equilibrium Bismarckian factor emerging in the hyperbolic voting game, and $\alpha_{TC}^{mv}(\tau)$, the equilibrium factor when $N^{y,TI} = N^{o,TI} = 0$. If hyperbolic individuals are able to form a coalition with exponential poor, the resulting pension system is more redistributive than that an ideal, exponential, one. The reasons for this excess of redistribution are quite intuitive: hyperbolic discounting introduces a third form of redistribution, besides from young to old and from rich to poor: from time consistent to hyperbolic individuals, or from individuals with long career to early retirees. This result is in line with the empirical evidence provided in Liberman (2001). In *(v)* we compare α^{opt} with the equilibrium one; we find that, while the former is increasing in N^{TI}, the latter is actually decreasing with it: if the number of hyperbolic individuals increases, their political power increases too, thus making the pension system more Beveridgean. The intuition for this result is simple: the social planner internalizes the negative externality between hyperbolic and exponential, and applies a corrective Pigouvian Tax, in the form of a higher Bismarckian factor. This tax takes into account into the budget constraint that present-biased workers will retire earlier: increasing α postpones workers' retirement.

8.5 Simultaneous Voting

The simultaneous equilibrium à la Shepsle is determined by aggregating the two reaction functions, $\tau_{TI}^{mv}(\alpha)$ and $\alpha_{TI}^{mv}(\alpha)$. The point(s) in which they intercepts, if any, is a candidate for being the equilibrium outcome of the simultaneous game, (α^s, τ^s). Depending on the values of the parameters and the income distribution, two equilibria are possible: if the income distribution is relatively skewed to the right, the resulting $\alpha^{mv}(\tau)$ is high. We refer to this equilibrium as a Bismarckian social security system. If the income distributions skewed to the left, we have a Beveridgean pension system (Figure 10).

Our objective is to show how time inconsistency in modifies the voting equilibrium: we then compare two situations: $(\alpha_{TI}^s, \tau_{TI}^s)$, the simultaneous equilibrium that arises in our hyperbolic model and $(\alpha_{TC}^s, \tau_{TC}^s)$, the equilibrium of an ideal, time consistent, economy. Therefore, in each picture, we depict four curves: the voting functions resulting from our hyperbolic model, $\tau_{TI}^{mv}(\alpha)$ and $\alpha_{TI}^{mv}(\tau)$, and the voting functions of the ideal time-consistent economy, $\tau_{TC}^{mv}(\alpha)$ and $\alpha_{TC}^{mv}(\tau)$. The intersections of these curves will give us two equilibrium points $(\alpha_{TI}^s, \tau_{TI}^s)$ and $(\alpha_{TC}^s, \tau_{TC}^s)$. From Proposition 6 we know that, when $\alpha \leq \frac{1}{2\delta}$, the overconfidence and the commitment effects go in opposite directions. In Figure 10, we have depicted the most interesting case, $\tau_{TI}^{mv}(\alpha) < \tau_{TC}^{mv}(\alpha)$. If a Bismarckian system emerges in equilibrium, by comparing $(\alpha_{TI}^s, \tau_{TI}^s)$ and $(\alpha_{TC}^s, \tau_{TC}^s)$, we can see that time inconsistency reduces the equilibrium value of α and increases the generosity of the social security with respect to the time consistent economy: hyperbolic agents are decisive in the political process and they can, at the same time, decrease τ and soften the link between pension benefit and working history, because it represents a way to raise old-age consumption. On the other hand, whenever a Beveridgean system emerges, redistribution is higher in the hyperbolic economy than in the exponential one.

Our theoretical results match are in line with the stylized facts presented in the introduction; first, we provide a political justification for the negative relationship between generosity of the social security system and degree of redistribution, as shown by Conde-Ruiz and Profeta (2005): in our equilibrium, Bismarckian systems are bigger than Beveridgean ones. Second, we have provided a possible explanation for the variability in the degree of redistribution observed in reality. In our model, four type of pension systems may emerge: Bismarckian with high level of redistributions (Belgium, Austria, Germany), that can be classified as "time inconsistent", and the winning coalition of hyperbolic agents decreases the level of α with respect to "time consistent" countries (Italy, Greece). The same happens with Beveridgean

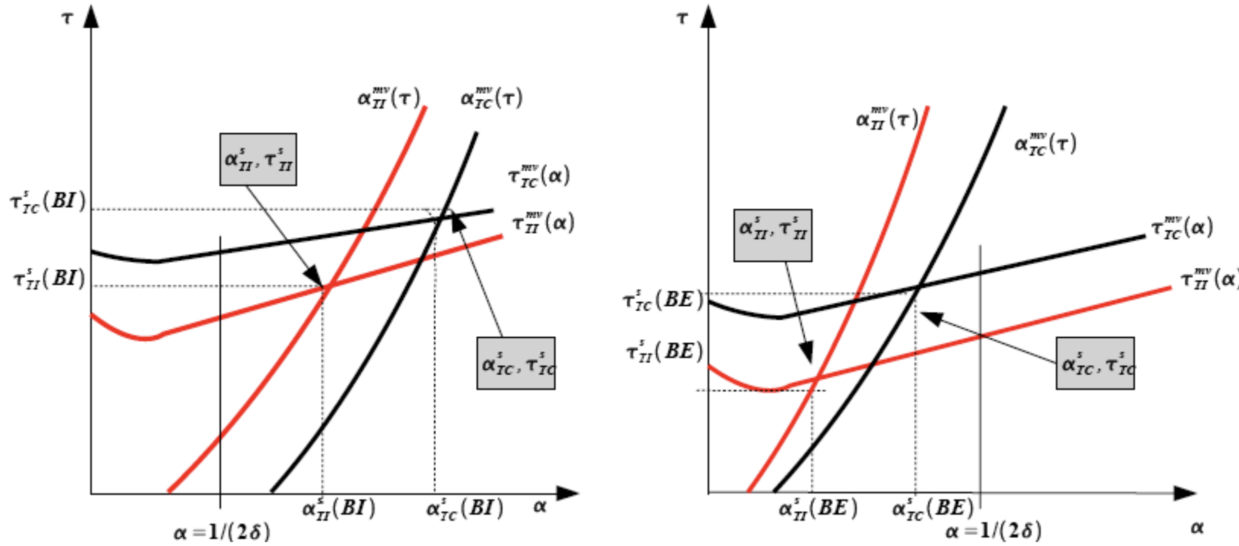

Figure 10: Simultaneous equilibrium in a Bismarckian (left) and Beveridgean system (right).

systems: from one side, we have hyperbolic countries (UK, New Zealand, Canada and Denmark) with a very high level of redistribution, and, on the other side, Beveridgean, time consistent, pension system with higher α (U.S., Japan, Switzerland).

9 Concluding Remarks

This paper studies a model of social security with endogenous retirement age and hyperbolic discounting in individuals' preferences.

Our model provides a justification for two observed stylized facts: the observed growth of voluntary early retirement and the drop in post-retirement consumption due to inadequate saving. We show that hyperbolic agents weights too much the costs associated to postponed retirement, *i.e.* foregone leisure, and too less the benefits, *i.e.* the increase in pension benefits. According to their preferences, early retirement is optimal. For the same reasons, an hyperbolic individual finds optimal to overconsume when young, but at the cost of a decrease in post-retirement consumption.

Our political model sheds light on three stylized facts, not yet addressed by the literature. First, as Liberman (2001) points out, redistribution in most pension system seems not to be related to lifetime income but to other factors: for instance, it goes from workers with longer careers to early retirees. Second, the classical distinction between Bismarckian and Beveridgean pension system, where the former are less redistributive than the latter, appears to be misleading, since we observe in reality very redistributive Bismarckian: other factors, besides income distribution, determine the ability of the system to redistribute income. Third, it exists a negative relationship between the size and degree of redistribution in most OECD pension systems. Bigger system (in terms of share of GDP devoted to pension transfer) are also the less redistributive.

We show that the winning coalition that determines the size and the degree of redistribution of the system always include hyperbolic individuals. More precisely, time inconsistent individuals prefer to decrease the size of the system, since a lower payroll tax represents for them a commitment device that increases both retirement age and savings. However, hyperbolic individuals prefer to increase the degree of redistribution of the system: besides the classical intragenerational and intergenerational redistribution, our model adds also redistribution from exponential to hyperbolic individuals. Finally, we show the majority voting equilibrium α is always lower than the one chosen by an utilitarian social planner. Whereas α^{opt} *increases* with the number of hyperbolic individuals, the equilibrium value of α is *decreasing* with it. The planner would like to provide a commitment device to hyperbolic workers, in the form of a tighter link between earnings and benefits, in order to force them to retire later. Therefore, whenever hyperbolic individuals have enough political power, the resulting pension system is small, *i.e.* low payroll tax but high redistribution.

The policy implications of our model are immediate: tightening the link between the length of the working career and benefits received, so that workers autonomously decide to retire later, can be an ineffective way to reduce government spending and the problem of early retirement. In this view, a "paternalistic" intervention, in the form of an increase of the minimal retirement age, appear to be more appropriate instrument to solve the pension crisis experienced by most European countries.

The next step of our research is to test empirically these predictions, and in particular to analyze whether the level of time inconsistency effectively differs among countries. If this is the case, than it would be relatively easier to compare if the presence of a high number of voluntary early retirees who regret the lack of accumulated saving leads to a particular pension system whose characteristics are in line with our predictions.

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Appendix

A Pension Programs in OECD Countries

Country	Type	Progressivity Index	Pension Expenditure (% GDP)
Australia	BE	74,8	4,7
Austria	BI	20,7	10,7
Belgium	BI	64,8	8,7
Canada	BE	86,5	4,8
Denmark	BE	91,7	8,3
France	BI	46,4	10,6
Germany	BI	22,9	11,7
Greece	BI	4,3	12,7
Ireland	BE	100	2,7
Italy	BI	4	11,3
Japan	BE	47,8	7,3
Netherlands	BE	5,7	6,4
New Zealand	BE	100	4,7
Spain	BI	13	8,3
Switzerland	BE	44,1	11,8
UK	BE	69,6	8,1
US	BE	40,6	5,3

Table V: Pension Programs in selected OECD countries (Disney 2001 and OECD 2005).

B Proofs

B.1 Proof of Proposition 1

Part *(ii)* of Lemma 1 show that $z_e > z_s$. To show the Pareto-improvement, we use the following notation. Let $U^o(z_j)$ the utility of a "pre-retirement" old, given its retirement age z_j. We want to show that:

$$f(\beta) = U(z_e) - U(z_s(\beta)) > 0$$

Note that $U(z_s(\beta))$ depends on β in two ways: β is the discount factor, so changes in β affect future consumption (when retired). Moreover, β enters into the expression that determines z_s itself. On the other hand, note that β only influences $U(z_e)$ through the first mechanism, as z_e does not depend on β. In the proof, we characterize the value of $f(.)$ in a neighborhood of $\beta = 1$. First, note that $f(1) = 0$, since

$z_e = z_s$. In order to evaluate the marginal variation of $f(.)$ around $\beta = 1$, we consider $f"(1)$ and $f'(1)$; we have:

$$f'(\beta) = \frac{\partial U(z_e)}{\partial \beta} - \frac{\partial U(z_s)}{\partial \beta} - \frac{\partial U(z_s)}{\partial z_s}\frac{dz_s}{d\beta}$$

Note that $f'(1) = 0$, as $\left.\frac{\partial U(z_e)}{\partial \beta}\right|_{\beta=1} = \left.\frac{\partial U(z_s)}{\partial \beta}\right|_{\beta=1}$ and $\left.\frac{\partial U(z_s)}{\partial z_s}\right|_{\beta=1} = \left.\frac{\partial U(z_e)}{\partial z_e}\right|_{\beta=1} = 0$.
Finally, it is possible to show that $f"(1) > 0$. Given that $f(1) = 0$, $f'(1) = 0$ and $f"(1) > 0$, there exists an interval $\left(\bar{\beta}, 1\right)$ such that $f(\beta) > 0$ for $\beta \in \left(\bar{\beta}, 1\right)$. This shows that a sophisticated worker is made better off by increasing its retirement age to z_e. Pareto dominance follows from the fact that all selves (pre and post-retirement) are made better off in two ways: first, they prefer z_e that influences pre and post retirement consumption and, second, they prefer gaining more pension benefits as implied by z_e.

B.2 Proof of Proposition 2

The proof for this proposition goes exactly like Proposition 1.

B.3 Proof of Lemma 2

From (17), the first derivative is:

$$\frac{\partial P(\widehat{z_j})(.)}{\partial \tau} = (1+n)(\alpha\omega + (1-\alpha)\bar{\omega}] + \theta\omega\alpha\left(z + \tau\frac{\partial \widehat{z_j}}{\partial \tau}\right)$$

A necessary and sufficient condition for this expression to be positive is the following:

$$2\theta^2\alpha\omega^2\tau(1-\hat{\beta}_j\delta\alpha) \leq \theta\alpha\omega^2 + (1+n)(\gamma - \hat{\beta}_j\psi\delta)(\alpha\omega + (1-\alpha)\bar{\omega})$$

and the definition of $\hat{\tau}$ follows immediately. Differentiating again $P(\widehat{z_j})$ with respect to τ gives us:

$$\frac{\partial^2 P(\widehat{z_j})(.)}{\partial \tau^2} = -\frac{2(1-\hat{\beta}_j\alpha\delta)}{\gamma - \hat{\beta}_j\psi\delta} < 0 \tag{27}$$

B.4 Proof of Proposition 3

The FOCs for the maximization problem for individuals of type j are:

$$-\omega u'\left(c_y^j\right) + \hat{\beta}_j\delta u'\left(c_o^j\right)\left[-\omega\theta\widehat{z_j} + \delta\frac{\partial P(.)}{\partial \tau}\right] + \lambda_\tau = 0 \tag{28}$$

$$-u'\left(c_y^j\right) + \hat{\beta}_j\delta u'\left(c_o^j\right) + \lambda_s = 0 \tag{29}$$

where λ_s and λ_τ are the Lagrange multipliers associated to the non-negativity constraints on s and τ. Depending on which constraints bind, we have different cases. Before doing that, we check whether the objective function $V^y(\tau_j^y, \alpha; \hat{\beta}_j, \omega)$ is concave in $(\tau_j^y, \widehat{s_j})$, so that the solutions represent indeed global optima. The Hessian matrix is:

$$D^2V^y(\tau_j^y, \widehat{s_j}) = \begin{bmatrix} \frac{\partial^2 V^y(\tau_j^y, \widehat{s_j})}{\partial(\tau_j^y)^2} & \frac{\partial^2 V^y(\tau_j^y, \widehat{s_j})}{\partial\tau_j^y\partial\widehat{s_j}} \\ \frac{\partial^2 V^y(\tau_j^y, \widehat{s_j})}{\partial\tau_j^y\partial\widehat{s_j}} & \frac{\partial^2 V^y(\tau_j^y, \widehat{s_j})}{\partial(\widehat{s_j})^2} \end{bmatrix}$$

where:

$$\frac{\partial^2 V^y(\tau_j^y,\widehat{s_j})}{\partial(\tau_j^y)^2} = \omega^2 u''\left(c_y^j\right) + \hat{\beta}_j \delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j} + \delta\frac{\partial P(\widehat{z_j})}{\partial \tau_j^y}\right]^2 + \hat{\beta}_j \delta u'\left(c_o^j\right)\left[-\omega\theta\widehat{z_j} + \delta\frac{\partial^2 P(\widehat{z_j})}{\partial\left(\tau_j^y\right)^2}\right] < 0 \tag{30}$$

$$\frac{\partial^2 V^y(\tau_j^y,\widehat{s_j})}{\partial(\widehat{s_j})^2} = u''\left(c_y^j\right) + \hat{\beta}_j \delta u''\left(c_o^j\right) < 0 \tag{31}$$

$$\frac{\partial^2 V^y(\tau_j^y,\widehat{s_j})}{\partial\tau_j^y \partial\widehat{s_j}} = u''\left(c_y^j\right)\omega + \hat{\beta}_j \delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j} + \delta\frac{\partial P(\widehat{z_j})}{\partial \tau_j^y}\right] \tag{32}$$

The determinant of the Hessian is:

$$\det(D^2 V^y(\tau_j^y,\widehat{s_j})) = \hat{\beta}_j \delta u''\left(c_y^j\right) u''\left(c_o^j\right)\left[\omega - \left(-\omega\theta\widehat{z_j} + \delta\frac{\partial P(\widehat{z_j})}{\partial \tau_j^y}\right)\right]^2 + \hat{\beta}_j \delta u'\left(c_o^j\right)\left(u''\left(c_y^j\right) + \hat{\beta}_j \delta u''\left(c_o^j\right)\right)\left[-\omega\theta\widehat{z_j} + \delta\frac{\partial^2 P(\widehat{z_j})}{\partial\left(\tau_j^y\right)^2}\right] \tag{33}$$

The sign of the determinant depends on that of the last term on the RHS of (33): by replacing (27) and (7) into (33), we find that a sufficient condition for (33) to be positive is $\hat{\beta} \leq \frac{1}{\alpha\delta}$, which is always satisfied. Since (30) and (31) are always negative, the Hessian matrix is negative definite and the objective function is concave in $(\tau_j^y, \widehat{s_j})$

Case 1: $\lambda_s = \lambda_\tau = 0$

None of the constraints is binding: replacing (29) into (28), we get:

$$\tau_y^+(\omega,\alpha;\hat{\beta}_j) = \frac{(\gamma - \hat{\beta}_j\delta\psi)[\omega - (1+n)(\alpha\omega + (1-\alpha)\bar{\omega}] + \omega\theta(1-\alpha)(\omega - \hat{\beta}_j\delta\psi)}{\theta^2\omega^2(1-\hat{\beta}_j\delta\alpha)(1-2\delta\alpha)} \tag{34}$$

The sign of this expression depends on the sign of the term $(1-2\delta\alpha)$. Since both cases are plausible, we discuss them separately. Suppose, first, that is negative ($\alpha\delta > 1/2$): a necessary and sufficient condition for this tax rate to be positive (part *(i)* of the proposition) is:

$$(\gamma - \hat{\beta}_j\delta\psi)[\omega - (1+n)(\alpha\omega + (1-\alpha)\bar{\omega}] + \omega\theta(1-\alpha)(\omega - \hat{\beta}_j\delta\psi) < 0$$

After some computations, we obtain the following second degree polynomial:

$$\omega^2 \underbrace{\left(\frac{\theta(1-\delta\alpha)}{\gamma - \hat{\beta}_j\delta\psi}\right)}_{a>0} + \omega \underbrace{\left(\delta(1+n)\alpha - \frac{\gamma - \hat{\beta}_j\delta\psi(1+\theta(1-\delta\alpha))}{\gamma - \hat{\beta}_j\delta\psi}\right)}_{b<0} - \underbrace{\delta(1-\alpha)(1+n)\bar{\omega}}_{c>0} < 0 \tag{35}$$

The solution has the form $\omega_1 \leq \omega \leq \tilde{\omega}$, where the threshold $\tilde{\omega}$ is given by $\tilde{\omega} = \frac{-b+\sqrt{b^2-4ac}}{2a}$; given that only the second root is positive, preferred tax rates are positive for $\omega_- \leq \omega \leq \tilde{\omega}$. To prove last part of *(i)*, differentiate $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$ with respect to ω :

$$\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\omega}=\frac{(\gamma-\hat{\beta}_j\delta\psi)(1-\alpha\delta(1+n))+\theta(1-\delta\alpha)(2\omega-\hat{\beta}_j\delta\psi)}{\theta^2\omega^2(1-\hat{\beta}_j\delta\alpha)(1-2\delta\alpha)}-\frac{2\,[\text{numerator of (34)}]}{\theta^2\omega^3(1-\hat{\beta}_j\delta\alpha)(1-2\delta\alpha)}$$

This expression is equivalent to:

$$\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\omega}=\frac{\delta(\gamma-\hat{\beta}_j\delta\psi)}{(1-\hat{\beta}_j\delta\alpha)(1-2\delta\alpha)}\left[\alpha(1+n)-1+2(1+n)(1-\alpha)\frac{\bar{\omega}}{\omega}+\frac{\hat{\beta}_j\psi\theta(1-\delta\alpha)}{\gamma-\hat{\beta}_j\delta\psi}\right] \tag{36}$$

The first term is negative, since $2\alpha\delta>1$, and a sufficient condition for the second term to be positive is:

$$\delta\alpha(1+n)+\frac{\hat{\beta}_j\delta\psi\theta(1-\delta\alpha)}{\gamma-\hat{\beta}_j\delta\psi}>1 \tag{37}$$

which is always verified since we impose $\gamma<\alpha\psi$. Thus, $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\omega}<0$, for $\omega_-<\omega<\tilde{\omega}$.

The impact of hyperbolic discounting on preferred tax rates (part *ii*) is given by:

$$\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\hat{\beta}_j}=\frac{(-\delta\psi)(\omega-\delta(1+n)[\alpha\omega+(1-\alpha)\bar{\omega}])-\delta\psi\omega\theta(1-\alpha\delta)}{\theta^2\omega^2(1-\hat{\beta}_j\delta\alpha)(1-2\delta\alpha)}+\frac{[\text{numerator of (34)}]\,(1-2\alpha\delta)\omega^2\theta^2}{\left[\theta^2\omega^2(1-\hat{\beta}_j\delta\alpha)(1-2\delta\alpha)\right]^2}$$

After some rearrangements, we get:

$$\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\hat{\beta}_j}=\underbrace{\frac{(\gamma\alpha-\psi)}{(1-2\alpha\delta)(1-\hat{\beta}_j\delta\alpha)}}_{>0}\left[1-\delta(1+n)\left(\alpha+\frac{\bar{\omega}}{\omega}\right)+\theta(1-\alpha\delta)(\alpha\omega-\psi)\right] \tag{38}$$

The sign of (38) depends on the sign of the term into square brackets, that is given by the following second degree inequality:

$$\theta(1-\alpha\delta)\alpha\omega^2-\underbrace{(1-\delta(1+n)-\theta(1-\alpha\delta))}_{b}\omega\underbrace{-\delta(1+n)\bar{\omega}}_{c}\gtreqless 0$$

This expression defines a threshold $\omega_b=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (the second root is negative, since $c<0$). It can be easily shown that $\omega_b<\tilde{\omega}$, and thus $\frac{\partial\tau^j}{\partial\hat{\beta}_j}\geq 0$ for $\breve{\omega}\leq\omega<\tilde{\omega}$ and $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\hat{\beta}_j}<0$ for $\omega^-\leq\omega<\breve{\omega}$.

Part *(iii)* of the proposition is obtained by differentiating $\tilde{\omega}$ with respect to $\hat{\beta}$.

$$\begin{aligned}\frac{\partial\tilde{\omega}}{\partial\hat{\beta}_j}= &\quad -(1+n)\delta\alpha+(\theta(1-\alpha\delta)+1)+\frac{1}{k}(1+n)\alpha\left\{2\left(\theta(1-\delta\alpha)+1\right)-\gamma\left(\theta(1-\delta\alpha)+2\right)\right\}+ \\ &\frac{1}{k}\left\{\left[\left(\theta(1-\delta\alpha)+1\right)\hat{\beta}_j\delta\psi-\gamma\right]\left(\theta(1-\delta\alpha)+1\right)-\left(\delta(1+n)\alpha\right)^2(\gamma-\hat{\beta}_j\delta\psi)-2\delta(1-\alpha)\bar{\omega}(1+n)\theta(1-\delta\alpha)\right\}\end{aligned} \tag{39}$$

where $k=\sqrt{\Delta\ of\ expression\ (35)}>0$. After some rearrangements, we obtain that the sign of $\frac{\partial\tilde{\omega}}{\partial\hat{\beta}_j}$ is equivalent to the sign of:

$$\begin{aligned}&\hat{\beta}_j\delta\psi\left[\delta(1+n)\alpha+(\theta(1-\delta\alpha)+1)\right]^2-\gamma\left[(1+n)\delta\alpha+1\right]^2- \\ &\gamma[\theta(1-\delta\alpha)\left[(1+n)\alpha\delta+1)\right]]-2\delta(1-\alpha)\bar{\omega}(1+n)\theta(1-\alpha\delta)\end{aligned}$$

The last two terms of this expression are negative: therefore, a sufficient condition for the whole expression to be negative is:

$$\hat{\beta}_j \delta\psi \left[\delta(1+n)\alpha + (\theta(1-\delta\alpha)+1)\right]^2 - \gamma\left[(1+n)\delta\alpha+1\right]^2 \leq 0$$

or:

$$\left[\sqrt{\hat{\beta}_j\delta\psi}((1+n)\delta\alpha + 1 + \theta(1-\delta\alpha)) - \sqrt{\gamma}\left[(1+n)\delta\alpha+1\right]\right] * \\ \left[\sqrt{\hat{\beta}_j\delta\psi}((1+n)\delta\alpha + (\theta(1-\delta\alpha)+1)) + \sqrt{\gamma}\left[(1+n)\delta\alpha+1\right]\right] \leq 0$$

Notice that the first term is always negative, since $\sqrt{\hat{\beta}_j\delta\psi} < \sqrt{\gamma}$ and $\theta(1-\delta\alpha) < 1$. Therefore, $\frac{\partial\tilde{\omega}}{\partial\hat{\beta}} < 0$.

Next step is to check how saving vary with productivity (part *iv*); by (29), we have:

$$\begin{aligned} \frac{\partial \widehat{s_j}(\omega)}{\partial\omega} = & -\frac{\left(-(1-\tau) + \omega\frac{\partial\tau}{\partial\omega}\right)u''(c_y^j)}{u''(c_y^j) + \hat{\beta}_j\delta u''(c_o^j)} + \\ & -\frac{\hat{\beta}_j\delta u''(c_o^j)\left[\widehat{z_j} - \tau(\widehat{z_j}\theta(1-\alpha\delta) - \delta(1+n)\alpha) - \frac{\partial\tau}{\partial\omega}\left[\omega\theta z(1-\alpha\delta) - \delta(1+n)\left((1-\alpha)\bar{\omega} + \alpha\omega\right)\right]\right]}{u''(c_y^j) + \hat{\beta}_j\delta u''(c_o^j)} \end{aligned} \tag{40}$$

The first term is positive, since $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\omega} < 0$ for $\omega\tilde{\omega}$. For the second term, we can find two sufficient conditions for this expression to be negative. We will see that this conditions are always satisfied, and then we conclude that $\frac{\partial\widehat{s_j}(\omega)}{\partial\omega} > 0$. These two conditions are:

1.
$$\widehat{z_j} - \tau_y^+(\omega,\alpha;\hat{\beta}_j)(\widehat{z_j}\theta(1-\alpha\delta) - \delta(1+n)\alpha) < 0$$

2.
$$\omega\theta\widehat{z_j}(1-\alpha\delta) + \delta(1+n)\left((1-\alpha)\bar{\omega} + \alpha\omega\right) < 0$$

The first condition can be rewritten as:

$$\tau > \frac{1}{\theta(1-\alpha\delta) - \frac{1}{\widehat{z_j}}\alpha\delta(1+n)} \Longleftrightarrow \alpha\delta > \frac{\theta-1}{\frac{1}{\widehat{z_j}}(\theta+1+n)}$$

Which is always satisfied, given that $\theta \in [0,1]$.

Replacing the expression for optimal retirement age (6), condition 2 can be rewritten as:

$$\omega^2\frac{(1-\alpha\delta)\theta(1-\tau\theta(1-\beta\delta\alpha)}{\gamma} - \delta(1+n)\alpha\omega - \delta(1+n)(1-\alpha)\bar{\omega} < 0 \Longleftrightarrow 0 \leq \omega \leq \check{\omega}$$

It can be easily checked that this threshold $\check{\omega}$ is always greater than $\tilde{\omega}$. Hence, $\frac{\partial\widehat{s_j}(\omega)}{\partial\omega} > 0, \forall\omega \leq \tilde{\omega}$. Moreover, from (29), we can see that the function $\widehat{s_j}(\omega)$ is negative when ω_- is low and positive when ω approaches to $\tilde{\omega}$. Thus, there exists a value $\omega' < \tilde{\omega}$ such that saving are zero for $\omega \leq \omega'$ and positive above. Therefore, all workers with income between ω' and $\hat{\omega}$ have an interior solution for both τ and $\widehat{s_j}$.

Part *(v)* of the proposition comes from noticing that, with $\tau_y^+(\omega,\alpha;\hat{\beta}_j)=1,\forall j$, (28) becomes:

$$-\omega u'(0)+\hat{\beta}_j\delta u'(c_o^j)\left[-\omega\theta\widehat{z_j}+\delta\frac{\partial P(\widehat{z_j})}{\partial\tau}\right]\Big|_{\tau=1}<0 \tag{41}$$

Since Inada conditions hold, we have: $\lim\limits_{x\to 0}u'(x)=+\infty$.

We know that for individuals with $\omega_-\leq\omega\leq\tilde{\omega}$, preferred tax rates $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$ are between 0 and 1. Therefore (part *vi*) we have:

$$\begin{aligned}\frac{\partial V^y\left(\tau_y^+(\omega,\alpha;\hat{\beta}_j),\alpha,\omega;\hat{\beta}_j\right)}{\partial\tau} &= 0 \\ \frac{\partial^2 V^y\left(\tau_y^+(\omega,\alpha;\hat{\beta}_j),\alpha,\omega;\hat{\beta}_j\right)}{\partial\tau^2} &< 0\end{aligned} \tag{42}$$

Differentiating the first expression with respect to α, we obtain:

$$\frac{\partial^2 V^y\left(\tau_y^+(\omega,\alpha;\hat{\beta}_j),\alpha,\omega;\hat{\beta}_j\right)}{\partial\tau^2}\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\alpha}+\frac{\partial^2 V^y\left(\tau_y^+(\omega,\alpha;\hat{\beta}_j),\alpha,\omega;\hat{\beta}_j\right)}{\partial\tau\partial\alpha}=0$$

The sign of $\frac{\partial\tau_y^+(.)}{\partial\alpha}$ is the same as $\frac{\partial V^y\left(\tau_y^+(\omega,\alpha;\hat{\beta}_j),\alpha,\omega;\hat{\beta}_j\right)}{\partial\tau\partial\alpha}$. From (23), we get:

$$\frac{\partial^2 V^y(.)}{\partial\tau\partial\alpha}=u''(c_o^j)\left(-\omega\theta\widehat{z_j}+\delta\frac{\partial P(.)}{\partial\tau}\right)\left(\frac{\partial c_o^j}{\partial\alpha}\right)+u'(c_o^j)\left(-\omega\theta\frac{\partial\widehat{z_j}}{\partial\alpha}+\delta\frac{\partial^2 P(\widehat{z_j})}{\partial\tau\partial\alpha}\right)$$

The first term is zero because of (28) (remember that we restrict consumption to be strictly positive). The second one is positive too; to see this, let us replace the expression for $\frac{\partial\widehat{z_j}}{\partial\alpha}$ and $\frac{\partial^2 P(\widehat{z_j})}{\partial\tau\partial\alpha}$. After some boring algebra:

$$\frac{\partial^2 V^y(.)}{\partial\tau\partial\alpha}=u'(c_o^j)\left[\delta(1+n)(\omega-\bar{\omega})+\frac{\theta\omega\delta}{\gamma-\hat{\beta}_j\delta\psi}\left(\omega-\hat{\beta}_j\delta\psi+\omega\theta\tau B\right)\right]$$

where $B=(4\delta\hat{\beta}_j\alpha-2-\hat{\beta}_j)$. For the whole second term to be positive, it suffices that $|B|<1$, which is always true. Finally, the sign of the entire expression depends crucially on the individual's income: if $\bar{\omega}\leq\omega\leq\tilde{\omega}$, $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\alpha}>0$. Otherwise, for very poor individuals, $\omega_-\leq\omega<<\bar{\omega}$, we have $\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\alpha}<0$.

Case 2: $\lambda_s>0,\ \lambda_\tau=0$

Individuals with earnings less than ω' choose a positive tax rate $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$ (given by 28) and no saving.

Case 3: $\lambda_s=0,\ \lambda_\tau>0$

Individuals with productivity above $\tilde{\omega}$ rely exclusively on private saving, that are defined by (29).

To determine the majority voting solution, we have to know how preferred tax rates $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$ vary with income for individuals below ω' (those with $\widehat{s_j}=0$). From (28):

$$\begin{aligned}\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\omega} &= -\frac{-u'\left(c_y^j\right)-\omega(1-\tau_y^+)u''\left(c_y^j\right)+\beta\delta u'\left(c_o^j\right)\left[-\theta\widehat{z_j}-\omega\theta\frac{\partial\widehat{z_j}}{\partial\omega}+\frac{\partial^2P(\widehat{z_j})}{\partial\tau_y^+\partial\omega}\right]}{D_{\tau_y^+}} \\ &+\frac{\beta\delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j}+\frac{\partial P(\widehat{z_j})}{\partial\tau_y^+}\right]\left[\widehat{z_j}(1-\theta\tau_y^+)+\delta\alpha\tau_y^+(1+n)+\delta\alpha\tau_y^+\widehat{z_j}\right]}{D_{\tau_y^+}}\end{aligned}$$

where $D_{\tau_y^+}$ is the second order derivative of $V^y(\tau_y^+,\widehat{s_j})$ with respect to τ_y^+, that is lower than zero, because:

$$\begin{aligned}\frac{\partial^2V^y(\tau_y^+,\widehat{s_j})}{\partial\left(\tau_y^+\right)^2} &= \omega^2u''\left(c_y^j\right)+\hat{\beta}_j\delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j}+\delta\frac{\partial P(\widehat{z_j})}{\partial\tau_y^+}\right]^2+ \\ &\hat{\beta}_j\delta u'\left(c_o^j\right)\left[-\omega\theta\frac{\partial\widehat{z_j}}{\partial\tau_y^+}+\delta\frac{\partial^2P(\widehat{z_j})}{\partial\left(\tau_y^+\right)^2}\right]+ \\ &\frac{\partial\widehat{s_j}}{\partial\tau_y^+}\left[\omega u''\left(c_y^j\right)+\hat{\beta}_j\delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j}+\delta\frac{\partial P(\widehat{z_j})}{\partial\tau_y^+}\right]\right]\end{aligned}$$

Replacing the expression for $\frac{\partial\widehat{s_j}}{\partial\tau_y^+}=-\frac{\omega u''\left(c_y^j\right)+\hat{\beta}_j\delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j}+\delta\frac{\partial P(\widehat{z_j})}{\partial\tau_y^+}\right]}{u''\left(c_y^j\right)+\hat{\beta}_j\delta u''\left(c_o^j\right)}$, and after some rearrangements, we obtain:

$$\begin{aligned}\frac{\partial^2V^j(\tau^{y,j},\widehat{s_j})}{\partial\left(\tau_y^+\right)^2} &= \frac{\hat{\beta}_j\delta u''\left(c_y^j\right)u''\left(c_o^j\right)\left[\omega-\left(-\omega\theta\widehat{z_j}+\delta\frac{\partial P(\widehat{z_j})}{\partial\tau_y^+}\right)\right]^2}{u''\left(c_y^j\right)+\hat{\beta}_j\delta u''\left(c_o^j\right)}+ \\ &\hat{\beta}_j\delta u'\left(c_o^j\right)\left[-\frac{\partial\widehat{z_j}}{\partial\tau_y^+}\theta\omega+\delta\frac{\partial^2P(\widehat{z_j})}{\partial\left(\tau_y^+\right)^2}\right] \qquad (43)\end{aligned}$$

where the second term on the right hand side is negative, as one can check by replacing expressions (7) and (27). Finally, we have:

$$\begin{aligned}\frac{\partial\tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial\omega} &= -\frac{-u'\left(c_y^j\right)(1-\epsilon)+\hat{\beta}_j\delta u'\left(c_o^j\right)\left[-\theta\widehat{z_j}-\omega\theta\frac{\partial\widehat{z_j}}{\partial\omega}+\frac{\partial^2P(\widehat{z_j})}{\partial\tau_y^+\partial\omega}\right]}{D_{\tau_y^+}}- \\ &\frac{\hat{\beta}_j\delta u''\left(c_o^j\right)\left[-\omega\theta\widehat{z_j}+\frac{\partial P(\widehat{z_j})}{\partial\tau_y^+}\right]\left[\widehat{z_j}(1-\theta\tau_y^+)+\delta\alpha\tau_y^+(1+n)+\delta\alpha\tau_y^+\widehat{z_j}\right]}{D_{\tau_y^+}}\end{aligned}$$

where ϵ is the coefficient of relative risk aversion, that is assumed to be lower than 1 (as in Casamatta, Cremer and Pestieau 2005). Notice that the first term in square brackets is negative because $\frac{\partial^2P(\widehat{z_j})}{\partial\tau_y^+\partial\omega}\leq 0$, and that the second term in square bracket is positive for $\omega\leq\omega'$. Thus, for individuals with income $\omega\leq\omega'$, who do not save privately, tax rates are decreasing with income.

B.5 Proof of Proposition 4

Case 1: $\lambda_s=\lambda_\tau=0$

We consider now the case $\alpha\delta<1/2$: $\tau_y^+(\omega,\alpha;\hat{\beta}_j)$ is positive only for $\tilde{\omega}\leq\omega\leq\omega_+$ (part i). For part *(ii)*, let us consider the expression (36). Now, the second term of the right hand side is positive and the first is positive for $\omega \geq \hat{\omega}$. Since $\hat{\omega} \geq \tilde{\omega}$, $\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \omega} > 0$.

For *(iii)*, we have that :

$$\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \hat{\beta}_j} = \underbrace{\frac{(\gamma\alpha-\psi)}{(1-2\alpha\delta)(1-\hat{\beta}_j\delta\alpha)}}_{<0}\left[1-\delta(1+n)\left(\alpha+\frac{\bar{\omega}}{\omega}\right)+\theta(1-\alpha\delta)(\alpha\omega-\psi)\right] \tag{44}$$

The sign of the derivative still depends on the sign of the term in square brackets. We have already shown (see Proposition 3) that it is positive for income levels above $\omega_b (< \tilde{\omega})$. It follows that for any ω such that $\tau_y^+(\omega,\alpha;\hat{\beta}_j) > 0$, we have that $\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \hat{\beta}_j} < 0$.

We have to check now that saving function is increasing with income for individuals with income greater than $\tilde{\omega}$:

$$\begin{aligned} \frac{\partial \widehat{s_j}(\omega)}{\partial \omega} \quad = \quad & -\frac{\left(-(1-\tau_y^+)+\omega\frac{\partial \widehat{z_j}}{\partial \omega}\right)u''\left(c_y^j\right)}{u''(c_y^j)+\hat{\beta}_j\delta u''(c_o^j)} \\ & -\frac{\hat{\beta}_j\delta u''(c_o^j)\left[\widehat{z_j}-\tau_y^+(\widehat{z_j}\theta(1-\alpha\delta)-\delta(1+n)\alpha)-\frac{\partial \tau_y^+(\omega,\alpha;\hat{\beta}_j)}{\partial \omega}[\omega\theta\widehat{z_j}(1-\alpha\delta)+\delta(1+n)((1-\alpha)\bar{\omega}+\alpha\omega)]\right]}{u''(c_y^j)+\hat{\beta}_j\delta u''(c_o^j)} > 0 \end{aligned} \tag{45}$$

The first term is negative and the second is positive: we are not able to give a clear sign to this expression: therefore, we assume (reasonably) that this derivative is positive. From (29) we can see that the function $\widehat{s_j}(\omega)$ is negative when ω_- is low and positive when ω approaches to $\tilde{\omega}$. Thus, there exists a value ω' such that saving are zero for $\omega \leq \omega'$ and positive above.

Parts *(iv)* and *(v)* follow from proposition 3. Part *(vi)* can be proved in the same way as in proposition 3.

Case 2: $\lambda_s > 0$, $\lambda_\tau > 0$

Individuals with productivity ω' prefer $\tau_y^+(\omega,\alpha;\hat{\beta}_j) = 0$ and no saving.

Case 3: $\lambda_s = 0$, $\lambda_\tau > 0$

Rich individuals with income $\omega' \leq \omega \leq \tilde{\omega}$ rely exclusively on private saving, whose expression is implicitly defined by (29).

B.6 Proof of Proposition 5

First, we show that the object function for old is concave: differentiating (24) twice with respect to τ_o :

$$\frac{\partial^2 V^o(\tau_o,\alpha;\beta_j,\omega)}{\partial\left(\tau_o\right)^2} = u'\left(c_o^j\right)\left[-\omega\theta z_j+\delta\frac{\partial^2 P(z_j)}{\partial(\tau_o)^2}\right]+u''\left(c_o^j\right)\left[-\omega\theta z_j+\hat{\beta}_j\delta\frac{\partial^2 P(z_j)}{\partial(\tau_o)^2}\right]^2 \tag{46}$$

From the discussion on (30), the first term in square brackets is negative and the objective function is concave.

To prove *(i)*, we differentiate $V^o(\tau_o,\alpha;\beta_j,\omega)$ at $\tau=0:$

$$\left.\frac{\partial V^o(\tau_o,\alpha;\beta_j,\omega)}{\partial \tau_o}\right|_{\tau=0} = u'\left(c_o^j\right)\left[-\omega\theta\left(\tfrac{\omega-\beta_j\delta\psi}{\gamma-\beta_j\delta\psi}\right)(1-\beta_j\delta\alpha)+\beta_j\delta(1+n)[\alpha\omega+(1-\alpha)\bar{\omega}]\right] \tag{47}$$

At $\tau=0$, everyone works in the second period. This expression is greater than 0 only for workers with income $\omega_- \leq \omega \leq \dot{\omega}$, where the threshold $\dot{\omega}$ is the (positive) root of the second-degree polynomial:

$$\omega^2\underbrace{\left(\frac{1}{\beta_j\delta}\right)}_{a} - \omega\underbrace{\left(\psi+\alpha(1+n)\frac{\theta\left(1-\beta_j\delta\alpha\right)}{\gamma-\beta_j\delta\psi}\right)}_{b} - \underbrace{\frac{(1+n)(1-\alpha)\bar{\omega}\theta\left(1-\beta_j\delta\alpha\right)}{\gamma-\beta_j\delta\psi}}_{c} < 0 \tag{48}$$

The solution is $0 \leq \omega \leq \dot{\omega} = \frac{-b+\sqrt{b^2-4ac}}{2a}$; $\tau_o^+(\omega,\alpha;\beta_j)$ is positive only for individuals with income below the threshold.

Part *(ii)* can be obtained by differentiating (28) with respect to ω, for $\omega \leq \dot{\omega}:$

$$\begin{aligned}\frac{d\tau_o^+(\omega,\alpha;\beta_j)}{d\omega} \quad = \quad & -\frac{\left[-\theta z_j(1-\beta_j\delta\alpha)-\omega\theta\frac{\partial z_j}{\partial\omega}+\beta_j\delta\alpha\left((1+n)+\theta z_j+\theta\omega\frac{\partial z_j}{\partial\omega}\right)\right]u'\left(c_o^j\right)}{D_\tau^2} \\ & +\frac{[-\omega\theta z_j(1-\beta_j\delta\alpha)+\beta_j\delta(1+n)(\alpha\omega+(1-\alpha)\bar{\omega})][z_j(1-\tau\theta)+\beta_j\delta(1+n)\alpha+\beta_j\delta\alpha\tau\theta z_j]u''\left(c_o^j\right)}{D_\tau^2}\end{aligned} \tag{49}$$

Where $D_\tau^2 < 0$ is the second derivative of the objective function. Given our assumptions on $u(.)$ and since we focus only on the case $\omega \leq \dot{\omega}$, the second term on the numerator is negative. For the first one, observe that:

$$-\theta z_j\left(1-\beta_j\delta\alpha\right)-\omega\theta\frac{\partial z_j}{\partial\omega}+\beta_j\delta\alpha\left((1+n)+\theta z_j+\theta\omega\frac{\partial z_j}{\partial\omega}\right)<0$$

$$\begin{aligned}\iff \quad & \omega^2\underbrace{\left(\frac{\theta\left(1-\beta_j\delta\alpha\right)\left(2-\tau\theta\left(1-\beta_j\delta\alpha\right)\right)}{\gamma-\beta_j\delta\psi}\right)}_{a} - \omega\underbrace{\left(\beta_j\delta\frac{\theta\psi\left(1-\beta_j\delta\alpha\right)}{\gamma-\beta_j\delta\psi}+\beta_j\delta\alpha(1+n)\right)}_{b} - \\ & \underbrace{\beta_j\delta(1+n)(1-\alpha)\bar{\omega}}_{c}\end{aligned} \tag{50}$$

The solution takes the form $0 \leq \omega \leq \ddot{\omega} = \frac{-b+\sqrt{b^2-4ac}}{2a}$. It can be shown that $\ddot{\omega} \leq \dot{\omega}$; thus, preferred tax rates are decreasing with income for $\ddot{\omega} \leq \omega \leq \dot{\omega}$. Unfortunately, we can not give clear predictions for productivity levels $\omega_- \leq \omega \leq \ddot{\omega}$: however, knowing that $\tau_o^+(\omega_-,\alpha;\beta_j) > 0$ (see part *(i)* of the proposition), we can compare the values of the preferred tax rates at ω_- and $\ddot{\omega}$; from equation (47), we can get the expression for preferred tax rate:

$$\tau_o^+(\omega,\alpha;\beta_j) = \frac{\omega\theta(\omega-\hat{\beta}_j\delta\psi)(1-\hat{\beta}_j\alpha\delta)-\hat{\beta}_j\delta(\gamma-\hat{\beta}_j\delta\psi)(1+n)(\alpha\omega+(1-\alpha)\bar{\omega})}{\omega^2\theta^2(1-\hat{\beta}_j\alpha\delta)} \tag{51}$$

It is easy to verify that $\tau_o^+(\omega_-,\alpha;\beta_j) < \tau_o^+(\ddot{\omega},\alpha;\beta_j)$; therefore, for $\omega_- \leq \omega \leq \ddot{\omega}$, optimal tax rates for old are increasing with income.

For *(iii)*, let us take the derivative of $\dot{\omega}$ with respect to $\beta_j:$

$$\frac{\partial\dot{\omega}}{\partial\beta_j} = \frac{\partial h}{\partial\beta_j} + \frac{1}{2\sqrt{\Delta \text{ of expression (48)}}}\left(\frac{\partial(h^2)}{\partial\beta_j} + \frac{\partial\left(\frac{(1-\alpha)\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)}{\theta(1-\beta_j\alpha\delta)\beta_j\delta}\right)}{\partial\beta_j}\right) \tag{52}$$

where $h=\frac{\psi}{2}+\frac{\alpha(1+n)(\gamma-\beta_j\delta\psi)}{2\theta(1-\beta_j\delta\alpha)}$. We want to show that all terms of (52) are negative: for the first one, note that:

$$\frac{\partial h}{\partial \beta_j}=\tfrac{2\alpha(1+n)\theta\delta(\alpha\gamma-\psi)}{[2\theta(1-\beta_j\delta\alpha)]^2}<0\Leftrightarrow \alpha\gamma<\psi$$

as we assume. The parenthesis is also negative, since:

$$\frac{\partial(h^2)}{\partial\beta_j}=2h\frac{\partial h}{\partial\beta_j}<0$$

and:

$$\frac{\partial\left(\frac{(1-\alpha)\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)}{\theta(1-\beta_j\alpha\delta)\beta_j\delta}\right)}{\partial\beta_j}=\tfrac{-\delta\psi(1+n)\bar{\omega}(1-\alpha)\theta(1-\beta_j\alpha\delta)\beta_j\delta-\theta\delta(1-\alpha)\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)(1-2\alpha\delta\beta_j)}{(\theta(1-\beta_j\alpha\delta)\beta_j\delta)^2}$$

This sign of this derivative depends crucially on the term $1-2\alpha\delta\beta$: for $\alpha\delta\leq 1/2$, the whole term is negative, and thus $\frac{\partial\dot{\omega}}{\partial\beta_j}<0$. For $\alpha\delta\geq 1/2$, rearranging the expression above, we get:

$$-(1+n)\bar{\omega}(1-\alpha)\theta\delta\left[-\gamma+\beta_j\psi\delta\frac{\beta_j\alpha\delta}{(2\alpha\delta\beta_j-1)}\right]<0$$

$$\Longleftrightarrow \gamma-\frac{\psi\beta_j^2\alpha\delta^2}{(2\alpha\delta\beta_j-1)}\leq 0$$

Since we assume γ being not too high, i.e. $\frac{\psi}{\alpha}>\gamma$, a sufficient condition for this inequality to be true is $\frac{\psi\beta_j^2\alpha\delta^2}{(2\alpha\delta\beta_j-1)}>\frac{\psi}{\alpha}\Longleftrightarrow(\beta_j\alpha\delta-1)^2>0$, which is always true. Therefore, also for $\alpha\delta\geq 1/2$, $\frac{\partial\dot{\omega}}{\partial\beta_j}<0$.

Part *(iv)* can be proved by differentiating (47) with respect to β_j :

$$\frac{d\tau_o^+(\omega,\alpha;\beta_j)}{d\beta_j}=\frac{u'\left(c_o^j\right)\left[-\omega\theta\frac{\partial z_j}{\partial\beta_j}+\delta\frac{\partial P(z_j)}{\partial\tau}+\beta_j\delta\frac{\partial^2P(z_j)}{\partial\tau\partial\beta_j}\right]}{D_\tau^2}$$

The sign of the derivative depends on the term in square brackets; replacing the expressions for $\frac{\partial z_j}{\partial\beta_j}$, $\frac{\partial P(z_j)}{\partial\tau}$ and $\frac{\partial^2P(z_j)}{\partial\tau\partial\beta_j}$, we get a second degree polynomial in ω such that $\frac{\partial\tau_o^+(\omega,\alpha;\beta_j)}{\partial\beta_j}\leq 0$ for $\omega_-\leq\omega\leq\omega_c$ and $\omega_d\leq\omega\leq\dot{\omega}$ and $\frac{\partial\tau_o^+(\omega,\alpha;\beta_j)}{\partial\beta_j}>0$ otherwise.

Finally, part *(v)* can be shown in the same way as Propositions 3 and 4: for individuals with $\omega_-\leq\omega\leq\dot{\omega}$, $\tau_o^+(\omega,\alpha;\beta_j)>0$. Therefore, we have:

$$\begin{aligned}\frac{\partial V^o\left(\tau_o^+(\omega,\alpha;\beta_j),\alpha,\omega;\beta_j\right)}{\partial\tau}&=0\\ \frac{\partial^2V^o\left(\tau_o^+(\omega,\alpha;\beta_j),\alpha,\omega;\beta_j\right)}{\partial\tau^2}&<0\end{aligned}$$

Differentiating the first expression with respect to α, we obtain:

$$\frac{\partial^2V^y\left(\tau_{yo}^+(\omega,\alpha;\beta_j),\alpha,\omega;\beta_j\right)}{\partial\tau^2}\frac{\partial\tau_o^+(\omega,\alpha;\beta_j)}{\partial\alpha}+\frac{\partial^2V^o\left(\tau_o^+(\omega,\alpha;\beta_j),\alpha,\omega;\beta_j\right)}{\partial\tau\partial\alpha}=0$$

The sign of $\frac{\partial\tau_o^+(.)}{\partial\alpha}$ is the same as $\frac{\partial^2V^o\left(\tau_o^+(\omega,\alpha;\beta_j),\alpha,\omega;\beta_j\right)}{\partial\tau\partial\alpha}$. From (24), we get:

$$\frac{\partial^2V^y(.)}{\partial\tau\partial\alpha}=u''(c_o^j)\left(-\omega\theta\widehat{z_j}+\delta\frac{\partial P(.)}{\partial\tau}\right)\left(\frac{\partial c_o^j}{\partial\alpha}\right)+u'(c_o^j)\left(-\omega\theta\frac{\partial\widehat{z_j}}{\partial\alpha}+\beta_j\delta\frac{\partial^2P(\widehat{z_j})}{\partial\tau\partial\alpha}\right)$$

The first term is zero because of the first order condition for τ. Moreover, replacing the expressions for $\frac{\partial \widehat{z_j}}{\partial \alpha}$ and $\frac{\partial^2 P(\widehat{z_j})}{\partial \tau \partial \alpha}$:

$$\frac{\partial^2 V^o\,(.)}{\partial \tau \partial \alpha} = u'(c_o^j)\left[\beta_j \delta (1+n)(\omega - \bar{\omega}) + \frac{\theta \omega \delta}{\gamma - \hat{\beta}_j \delta \psi}\left(\omega - \hat{\beta}_j \delta \psi + \omega \theta \tau B\right)\right]$$

where $B = \delta \beta_j \alpha (1 + 3\beta_j) - 1 - \beta_j - \beta_j^2 > 0$. Therefore, this term (and also $\frac{\partial \tau_o^+(\omega, \alpha; \beta_j)}{\partial \alpha}$) is positive for $\bar{\omega} \leq \omega \leq \tilde{\omega}$. Otherwise, for very poor individuals, $\omega_- \leq \omega << \bar{\omega}$, we have $\frac{\partial \tau_o^+(\omega, \alpha; \beta_j)}{\partial \alpha} < 0$.

B.7 Proof that $\tau_o^+(\ddot{\omega}, \alpha; \beta_j)$ and $\tau_y^+(\omega_-, \alpha; \hat{\beta}_j))$ are not comparable

When $\alpha\delta > 1/2$, the maximal tax rate for old is given by (51), while the maximal rate for young is (34). To determine which tax rate prevails, we have to find the sign of the following expression:

$$sign\left\{\theta\left[\frac{\ddot{\omega} - \hat{\beta}_j \delta \psi}{\ddot{\omega}} - \frac{\omega_- - \hat{\beta}_j \delta \psi}{\omega_-(1-2\alpha\delta)} - \frac{(\gamma - \hat{\beta}_j \delta \psi)}{\omega_-(1-2\alpha\delta)}\right] - (\gamma - \hat{\beta}_j \delta \psi)\left[\frac{\hat{\beta}_j \delta (1+n)(\alpha \ddot{\omega}_- + (1-\alpha)\bar{\omega})}{\ddot{\omega}^2(1-\beta\alpha\delta)} - \frac{\delta(1+n)(\alpha\omega_- + (1-\alpha)\bar{\omega})}{\omega_-^2(1-2\alpha\delta)}\right]\right\}$$

The first term is positive, but the second is negative: a deeper analysis does not provide any interesting conditions for the whole expression to have a clear sign.

B.8 Proof that Preferences over α are Single Crossing

Single crossing requires that, for τ_j^i $i = y, o$ fixed, $\omega_1 < \omega_2$ and $\alpha_1 < \alpha_2$:

$$V^y(\tau, \alpha_2; \hat{\beta}_j, \omega_1) > V^y(\tau, \alpha_1; \hat{\beta}_j, \omega_1) \Longrightarrow V^y(\tau, \alpha_2; \hat{\beta}_j, \omega_2) > V^y(\tau, \alpha_1; \hat{\beta}_j, \omega_2)$$

Let us assume, without loss of generality, that $V^y(\tau, \alpha_2; \hat{\beta}_j, \omega_1) > V^y(\tau, \alpha_1; \hat{\beta}_j, \omega_1)$; it follows that:

$$[\widehat{z_j}(\alpha_2, \omega_1) - \widehat{z_j}(\alpha_1, \omega_1)]\,[\omega_1(1-\tau\theta) - (\gamma + \delta\psi)\,(\widehat{z_j}(\alpha_2, \omega_1) + \widehat{z_j}(\alpha_1, \omega_1)) - \delta\psi] +$$

$$+\delta\tau\omega_1\,[(1+n)(\alpha_2 - \alpha_1)] + \theta\,[\alpha_2 \widehat{z_j}(\alpha_2, \omega_1) - \alpha_1 \widehat{z_j}(\alpha_1, \omega_1)] > \bar{\omega}\delta\tau(1+n)(\alpha_2 - \alpha_1)$$

After having replaced for the expressions for $\widehat{z_j}(\alpha_2, \omega_1)$ and $\widehat{z_j}(\alpha_1, \omega_1)$, we get:

$$\omega_1 \underbrace{\left\{(1-\tau\theta)\left[\left(\gamma - \hat{\beta}_j \delta \psi\right)(1+\hat{\beta}_j) - \hat{\beta}_j\right] + \frac{\delta\theta\tau\hat{\beta}_j(\alpha_2+\alpha_1)(\gamma(2-\hat{\beta}_j) - 3\psi\delta\hat{\beta}_j)}{2}\right\}}_{K} + A > \frac{\bar{\omega}(1+n)(\gamma - \hat{\beta}_j \delta \psi)^2}{\theta \omega_1}$$

where $A = (1+n)(\gamma - \hat{\beta}_j \delta \psi)^2 - \theta \hat{\beta}_j \delta \psi (\gamma - \delta\psi)(1 + 2\hat{\beta}_j)$ is a constant term that does not depend on income. Given our initial assumption $\omega_1 < \omega_2$, we have:

$$\omega_2 K > \omega_1 K > \frac{\bar{\omega}(1+n)(\gamma - \hat{\beta}_j \delta \psi)^2}{\theta \omega_1} > \frac{\bar{\omega}(1+n)(\gamma - \hat{\beta}_j \delta \psi)^2}{\theta \omega_2}$$

which is exactly what we wanted to show.

B.9 Proof of Proposition 8

For part *(i)*, maximization of (25), give us the following FOC, evaluated at $\alpha = 0:$

$$\left.\frac{\partial V_j^o(.)}{\partial \alpha}\right|_{\alpha=0} : u'(c_o)\left[\omega(1-\tau\theta) - \gamma z_j \frac{\partial z_j}{\partial \alpha} + \beta_j \delta\left(\tau\omega(1+n+\theta z_j) + \alpha\tau\omega\theta\frac{\partial z_j}{\partial \alpha} - \bar{\omega}(1+n)\tau - \psi(1-z_j)\frac{\partial z_j}{\partial \alpha}\right)\right] = 0 \tag{53}$$

After some rearrangements:

$$\omega^2\underbrace{\theta(1-\tau\theta)}_{a} + \omega\underbrace{[(1+n)(\gamma-\beta_j\delta\psi) - \beta_j\theta\delta\psi]}_{b} - \underbrace{\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)}_{c}$$

We get a second degree polynomial, whose roots are given by $\omega_{e',e} = \frac{-b\pm\sqrt{b^2+4ac}}{2a}$. Therefore, $\alpha_o^+(\tau,\omega;\beta_j) > 0$ for income levels $\omega_e \le \omega \le \omega_+$, whereas, for productivity $\omega_- \le \omega \le \omega_f$, $\alpha_o^+(\tau,\omega;\beta_j) = 0$. In particular, $\omega_f = \frac{-((1+n)(\gamma-\beta_j\delta\psi)-\beta_j\theta\delta\psi)+\sqrt{((1+n)(\gamma-\beta_j\delta\psi)-\beta_j\theta\delta\psi)^2+4\theta(1-\tau\theta)\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)}}{2\theta(1-\tau\theta)}$.

To show part *(ii)* of the proposition, we derive the threshold ω_f with respect to β_j:

$$\frac{\partial\omega_f}{\partial\beta_j} = \frac{(1+n)\delta\psi+\theta\delta\psi}{2\theta(1-\tau\theta)}\left(1 - \frac{((1+n)(\gamma-\beta_j\delta\psi)-\beta_j\theta\delta\psi)}{\sqrt{X}}\right) - \frac{4\theta(1-\tau\theta)\bar{\omega}(1+n)\delta\psi}{\sqrt{X}}$$

where $X = ((1+n)(\gamma-\beta_j\delta\psi) - \beta_j\theta\delta\psi)^2 + 4\theta(1-\tau\theta)\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)$. Notice that:

$$\frac{((1+n)(\gamma-\beta_j\delta\psi)-\beta_j\theta\delta\psi)^2}{((1+n)(\gamma-\beta_j\delta\psi)-\beta_j\theta\delta\psi)^2+4\theta(1-\tau\theta)\bar{\omega}(1+n)(\gamma-\beta_j\delta\psi)} < 1$$

Therefore, $\frac{\partial\omega_f}{\partial\beta_j} < 0$.

For part *(iii)*, we use the implicit function theorem to obtain:

$$\frac{\partial\alpha_o^+(\tau,\omega;\beta_j)}{\partial\hat{\beta}_j} = -\frac{u'(c_o)\left[\frac{\partial^2 z_j}{\partial\alpha\partial\beta_j}(\omega(1-\tau\theta)-z_j(\gamma-\beta_j\delta\psi)+\beta_j\delta\alpha\tau\omega\theta-\beta_j\delta\psi) - \frac{\partial z_j}{\partial\beta_j}\left(\frac{\partial z_j}{\partial\alpha}(\gamma-\beta_j\delta\psi)-\beta_j\delta\tau\omega\theta\right)+\frac{\delta}{\beta_j}\frac{\partial z_j}{\partial\alpha}(\gamma z_j-\omega(1-\tau\theta))\right]}{D_\alpha^2}$$

Replacing the expressions for $\frac{\partial^2 z_j}{\partial\alpha\partial\beta_j}, \frac{\partial z_j}{\partial\beta_j}, \frac{\partial z_j}{\partial\alpha}$ and z_j, we get:

$$\frac{\partial\alpha_o^+(\tau,\omega;\beta_j)}{\partial\hat{\beta}_j} = -\frac{u'(c_o)\delta^2\frac{\partial z_j}{\partial\alpha}(\gamma(\alpha\tau\omega\theta-\psi)+\omega(1-\tau\theta)\psi)}{D_\alpha^2} > 0$$

Since $D_\alpha^2 < 0$ and the numerator is always positive.

For *(iii)*, we differentiate, with respect to α, $\frac{\partial V_j^o(.)}{\partial\alpha} = 0:$

$$\frac{\partial^2 V^o((\alpha_o^+(\omega,\tau;\beta_j),\tau,\omega;\beta_j))}{\partial\alpha^2}\frac{\partial\alpha_o^+(\tau,\omega;\beta_j)}{\partial\tau} + \frac{\partial^2 V^o((\alpha_o^+(\omega,\tau;\beta_j),\tau,\omega;\beta_j)}{\partial\tau\partial\alpha} = 0$$

Therefore, $\frac{\partial\alpha_o^+(\tau,\omega;\beta_j)}{\partial\tau}$ has the same sign as $\frac{\partial^2 V^o(.)}{\partial\tau\partial\alpha}$. Following our previous discussion, we have that for productivity above the average, $\frac{\partial\alpha_o^+(\tau,\omega;\beta_j)}{\partial\tau} > 0$.

B.10 Proof of Proposition 9

For part *(i)*, maximization of (25), give us the following FOC, evaluated at $\alpha = 0$:

$$\left.\frac{\partial V_j^o(.)}{\partial \alpha}\right|_{\alpha=0} : u'(c_o)\left[\omega(1-\tau\theta) - \gamma \hat{z}_j \frac{\partial \hat{z}_j}{\partial \alpha} + \delta\left(\tau\omega(1+n+\theta\hat{z}_j) + \alpha\tau\omega\theta\frac{\partial \hat{z}_j}{\partial \alpha} - \bar{\omega}(1+n)\tau - \psi(1-\hat{z}_j)\frac{\partial \hat{z}_j}{\partial \alpha}\right)\right] = 0$$

After some rearrangements, we have that young vote have a preferred positive α if and only if:

$$\omega^2 \underbrace{\theta(1-\tau\theta)\left(\gamma - \hat{\beta}_j^2\delta\psi\right)}_{a} - \omega \underbrace{\left[(1+n)\left(\gamma - \hat{\beta}_j\delta\psi\right) + \frac{\hat{\beta}_j\theta\delta\psi}{(\gamma - \hat{\beta}_j\delta\psi)}\left(\gamma(2-\hat{\beta}_j) + \hat{\beta}_j\delta\psi\right)\right]}_{b} - \underbrace{\bar{\omega}(1+n)\left(\gamma - \hat{\beta}_j\delta\psi\right)}_{c} > 0$$

We get a second degree polynomial, whose roots are given by $\omega_{f',f} = \frac{-b\pm\sqrt{b^2+4ac}}{2a}$. Being the first root negative, $\alpha_o^+(\tau,\omega;\beta_j) > 0$ only for $\omega_f \leq \omega \leq \omega_+$. To prove parts *(ii)* and *(iii)*, we proceed as in Proposition 8.